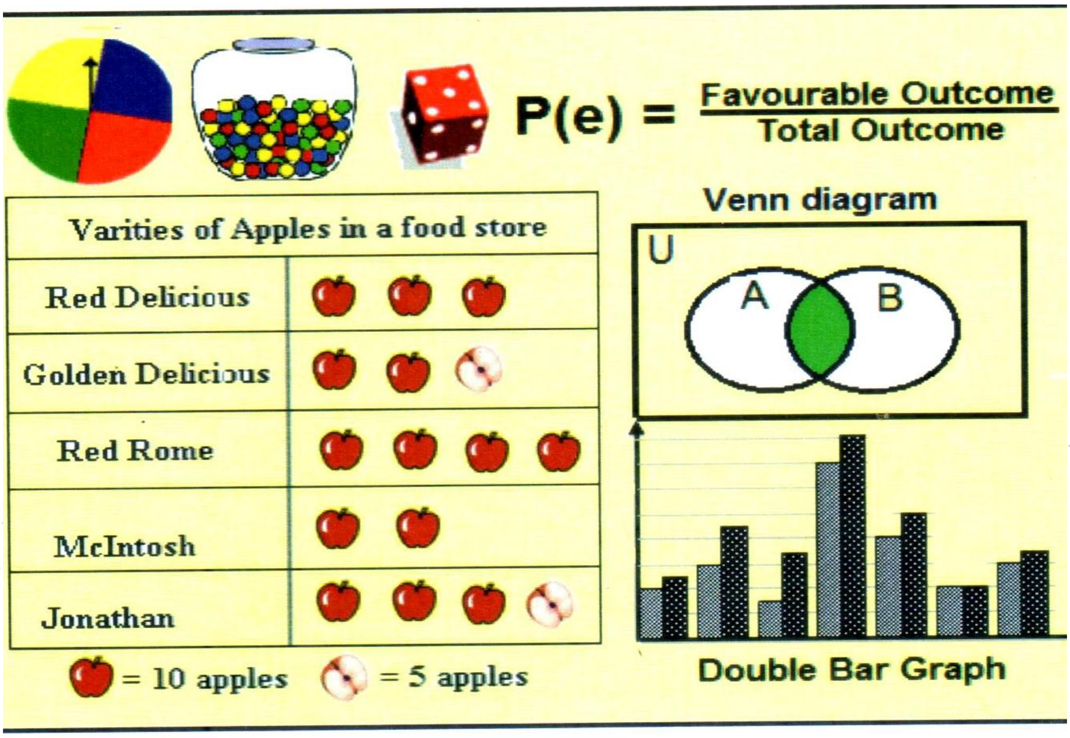
DEPARTMENT OF EDUCATION

GRADE 8

MATHEMATICS

STRAND 5



DATA AND CHANCE

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PAPUA NEW GUINEA

GRADE 8

MATHEMATICS

STRAND 5

CHANCE AND DATA

SUB-STRAND 1:	**GRAPHS**
SUB-STRAND 2:	**STATISTICS**
SUB-STRAND 3:	**SETS**
SUB-STRAND 4:	**PROBABIBILITY**

Acknowledgements

We acknowledge the contributions of all Secondary and Upper Primary Teachers who in one way or another helped to develop this Course.

Special thanks is given to the Staff of the Mathematics Department of FODE who played active role in coordinating writing workshops, outsourcing lesson writing and editing processes, involving selected teachers of NCD.

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Principal- FODE

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SECRETARY'S MESSAGE

Achieving a better future by individual students and their families, communities or the nation as a whole, depends on the kind of curriculum and the way it is delivered.

This course is part and parcel of the new reformed curriculum. The learning outcomes are student-centered with demonstrations and activities that can be assessed.

It maintains the rationale, goals, aims and principles of the national curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision by Flexible, Open and Distance Education as an alternative pathway of formal education.

The course promotes Papua New Guinea values and beliefs which are found in our Constitution and Government Policies. It is developed in line with the National Education Plans and addresses an increase in the number of school leavers as a result of lack of access to secondary and higher educational institutions.

Flexible, Open and Distance Education curriculum is guided by the Department of Education‟s Mission which is fivefold:

- to facilitate and promote the integral development of every individual
- to develop and encourage an education system that satisfies the requirements of Papua New Guinea and its people
- to establish, preserve and improve standards of education throughout Papua New Guinea
- to make the benefits of such education available as widely as possible to all of the people
- to make the education accessible to the poor and physically, mentally and socially handicapped as well as to those who are educationally disadvantaged.

The college is enhanced through this course to provide alternative and comparable pathways for students and adults to complete their education through a one system, two pathways and same outcomes.

It is our vision that Papua New Guineans‟ harness all appropriate and affordable technologies to pursue this program.

I commend all the teachers, curriculum writers and instructional designers who have contributed towards the development of this course.

………………………………………

DR. UKE KOMBRA, PhD
Acting Secretary for Education

STRAND 5: DATA AND CHANCE

Introduction

Dear Student,

This is the Fifth Strand of the Grade 8 Mathematics Course. It is based on the NDOE Upper Primary Mathematics Syllabus and Curriculum framework for Grade 8.

This Strand consists of four Sub-strands:

Sub-strand 1: Graphs
Sub-strand 2: Statistics
Sub-strand 3: Sets
Sub-strand 4: Probability

Sub-strand 1 – **Graphs** – You will present data in tables and graphs and interpret different types of graphs

Sub-strand 2 – **Statistics** – You will identify the measures of central tendency, calculate the three measures of central tendency, differentiate a grouped frequency distribution from an ungrouped frequency distribution, draw a frequency distribution table from a set of data and interpret the resulting histograms accurately.

Sub-strand 3 – **Sets** – You will explore empty sets, intersection and union of sets, use a variety of classification methods, use Venn diagrams to illustrate classifications and apply sets in solving problems from real life.

Sub-strand 4 – **Probability** – You will explore the results of independent events, calculate probabilities from individual event probabilities and explore the chances of winning or losing a game.

You will find that each lesson has reading materials to study, worked examples to help you, and a Practice Exercise. The answers to practice exercises are given at the end of each sub-strand.

All the lessons are written in simple language with comic characters to guide you and many worked examples to help you. The practice exercises to help you to learn the process of working out problems.

We hope you enjoy using this Stand.

All the best!

Mathematics Department
FODE

STUDY GUIDE

Follow the steps given below as you work through the Strand.

Step 1: Start with SUB-STRAND 1 Lesson 1 and work through it.

Step 2: When you complete Lesson 1, do Practice Exercise 1.

Step 3: After you have completed Practice Exercise 1, check your work. The answers are given at the end of the SUB-STRAND 1.

Step 4: Then, revise Lesson 1 and correct your mistakes, if any.

Step 5: When you have completed all these steps, tick the check-box for the Lesson, on the Contents Page (page 3) Like this:

$\boxed{\surd}$ Lesson 1: Pictograms

Then go on to the next Lesson. Repeat the process until you complete all of the lessons in Sub-strand 1.

> As you complete each lesson, tick the check-box for that lesson, on the Content"s page 3, like this $\boxed{\surd}$.This helps you to check on your progress.

Step 6: Revise the Sub-strand using Sub-strand 1 Summary, then do Sub-strand Test 1 in Workbook 5.

Then go on to the next Sub-strand. Repeat the process until you complete all of the four Sub-strands in Strand 5.

Assignment: (Four Sub-strand Tests and a Strand Test)
When you have revised each Sub-strand using the Sub-strand Summary, do the Sub-strand Test for that Sub-strand in your Assignment Book. The Course book tells you when to do each Sub-strand Test.

When you have completed the four Sub-strand Tests, revise well and do the Strand Test. The Assignment Book tells you when to do the Strand Test.

> The Sub-strand Tests and the Strand Test in the Assignment will be marked by your Distance Teacher. The marks you score in each Assignment Book will count towards your final mark. If you score less than 50%, you will repeat that Assignment Book.

Remember, if you score less than 50% in three Assignments, your enrolment will be cancelled. So, work carefully and make sure that you pass all the Assignments.

SUB STRAND 1

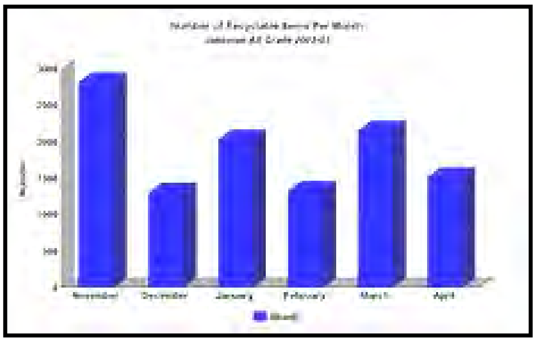
GRAPHS

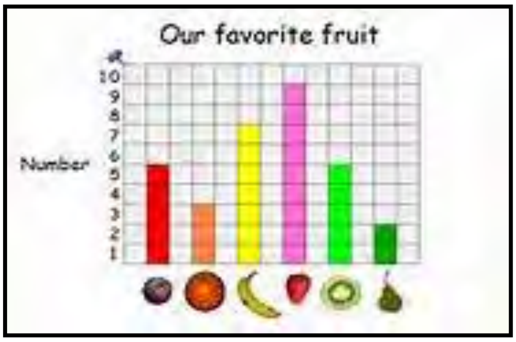
SUB-STRAND 1: GRAPHS

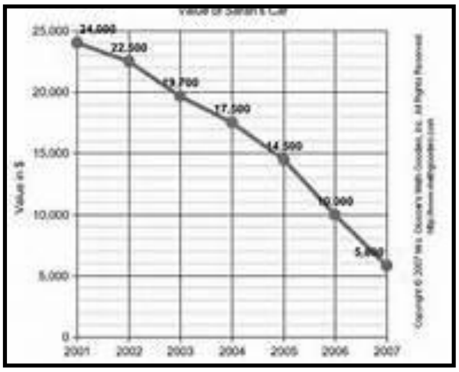
Introduction

Welcome to your Grade 8 Mathematics Strand 5, Sub-strand 1, in this sub-strand you will extend your knowledge on graphs by learning interesting and useful information about graphs.

Below are examples of graphs.







Graphs and Charts are well-organized methods for displaying information in a simple manner. Using this form of representation helps viewers to understand and interpret the information more easily and efficiently. For this reason, many statisticians and researchers these days see the importance of graphs to display information so that it is easy to interpret.

We will learn to do the same.

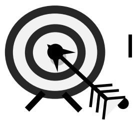
This Sub-strand will help you:

- present collated data in tables and graphs
- interpret correctly information presented in a variety of statistical graphs
- interpret and solve information from tables and graphs.

We hope you enjoy this sub strand.

Lesson 1: Pictograms

You learnt something about pictographs in your Grade 7 Mathematics Strand 5 Sub-strand 1 Lesson 2.



In this lesson, you will

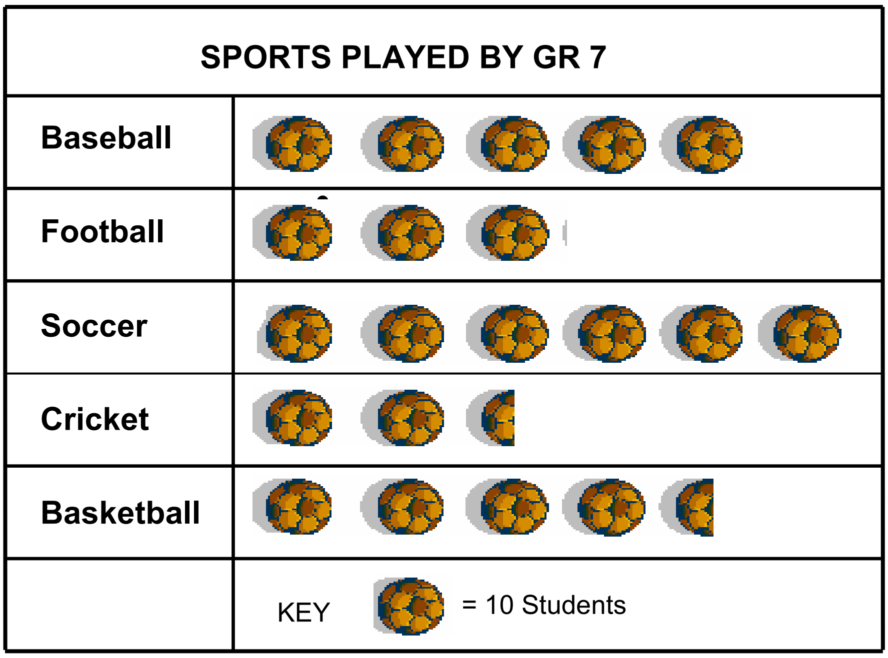
- describe pictographs
- draw a pictograph using given information
- solve problems related to pictographs.

In Grade 7, we learnt that statistical data can be graphically presented according to some scale or level of measurement. A graphic presentation shows numerical data in pictures such as a pictograph, a bar graph, a line graph and a circular or a pie graph.

For this section, we are going to deal with pictographs. You have studied pictographs in Grade 7.

First, let us look at this information.

Figure 1.1



Charles made a report on the number of students in Grade 7 playing the different kinds of sports. He thought about impressing his teacher so, to do the trick, he decided to draw pictures of balls to represent students playing the sport. His graph looks like the above.

We name the graph a **picture graph**.

A picture graph (also known as a pictograph) is one of the simplest types of graphs. Picture graphs convey information by using relevant pictures as symbols to represent quantities.

Here are other examples.

1. One stick man might represent one million people in a display of population statistics.

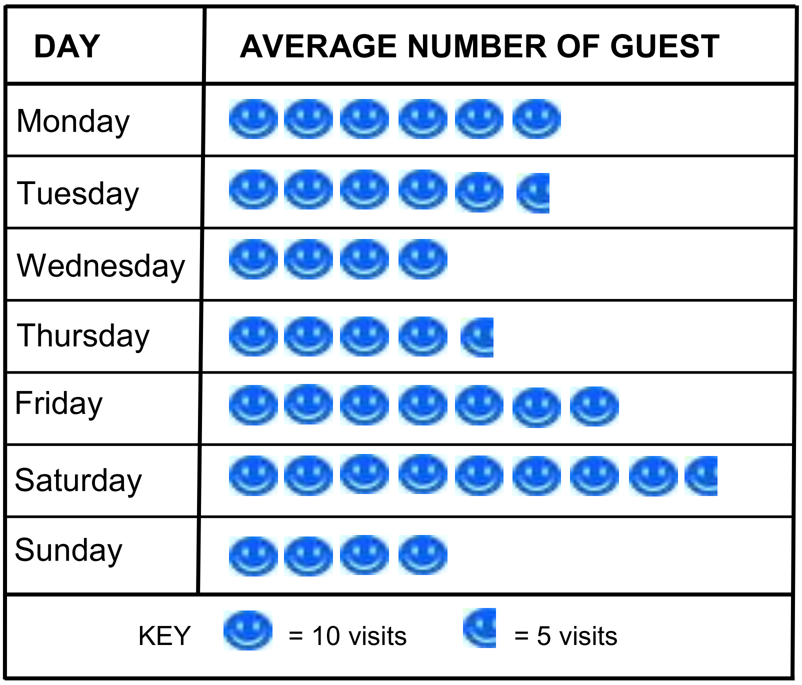
2. A single car symbol might represent ten vehicles in a survey of vehicle types owned by families at a school.

A key

A **key** is used to show the quantity of each feature that each symbol represents. Half a symbol would indicate half of the given amount. A complete pie might represent 20 pies so a quarter of a pie would indicate 5 pies.

Example 1

Figure 1.2



In this picture graph, the  symbol has been used to show 10 guests, while  has been used to indicate 5 guests.

Using the key provided, we know that 60 guests stayed at the motel on Monday night and 55 guests stayed on Tuesday night.

Next we are going to draw pictographs using given information.

Here is an example.

Table 1.1

OSC LTD DIRECTORSHIP ELECTION RESULTS

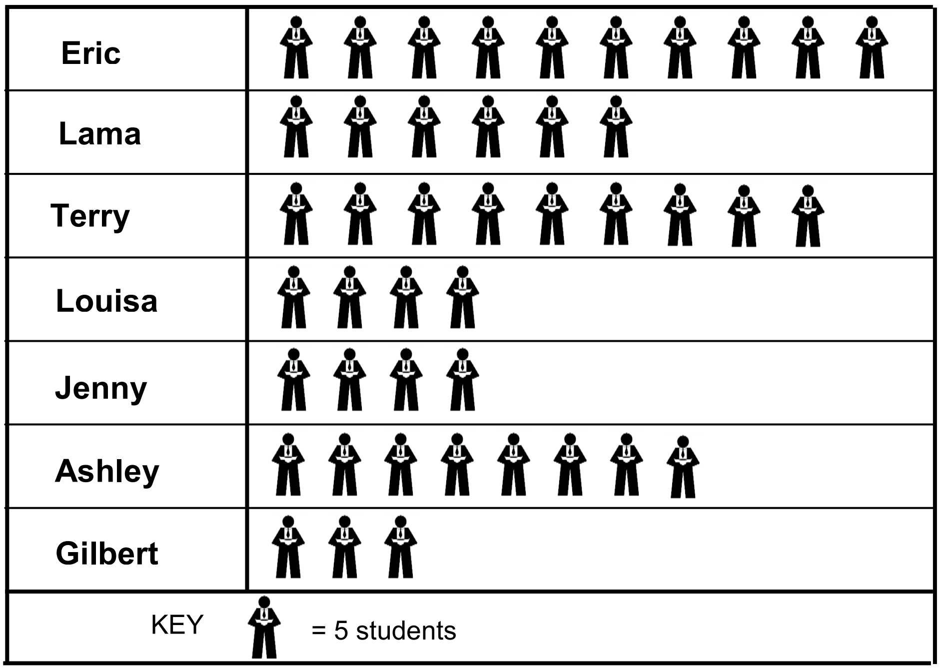
Candidates	Total Vote Garnered
Eric	50
Lama	30
Terry	45
Louisa	20
Jenny	20
Ashley	40
Gilbert	15

Use the data in the table above. Draw your pictograph by following these steps.

1. Decide on the symbol to use.
2. Decide on the scale by figuring out how many votes one symbol will represent. Do you need to use a part of a symbol to represent a group smaller than what a whole symbol represents?
3. Draw the graph. Repeat the symbols as many times as needed to show the size of each group.
4. Write the scale (Key) used and the title of the graph.

Solution: This is what your picture graph will look like.

OSC LTD DIRECTORSHIP ELECTION RESULTS



Note that in the picture graph, a picture of a man  has been used to represent the number of votes for each candidate. The man or any symbol chosen is the scale.

Now, understanding and interpreting the pictograph, answer the following questions.

1. What does the graph represent?
2. How many votes did the candidates receive?
3. Where there candidates who got equal number of votes?
4. Who was elected director?
5. How many employees voted?
6. By how many votes did Eric win?

Your answers must be similar to the ones below.

1. The graph represents the result of the election for the OSC Ltd Directorship.
2. The number of votes that each candidate received is as follow: Eric, 5; Lama, 30; Terry, 45; Louisa, Jenny, 20; Ashley, 40 and Gilbert, 15.
3. Yes,
4. Eric
5. 200 employees
6. 5 votes

NOW DO PRACTICE EXERCISE 1

Practice Exercise 1

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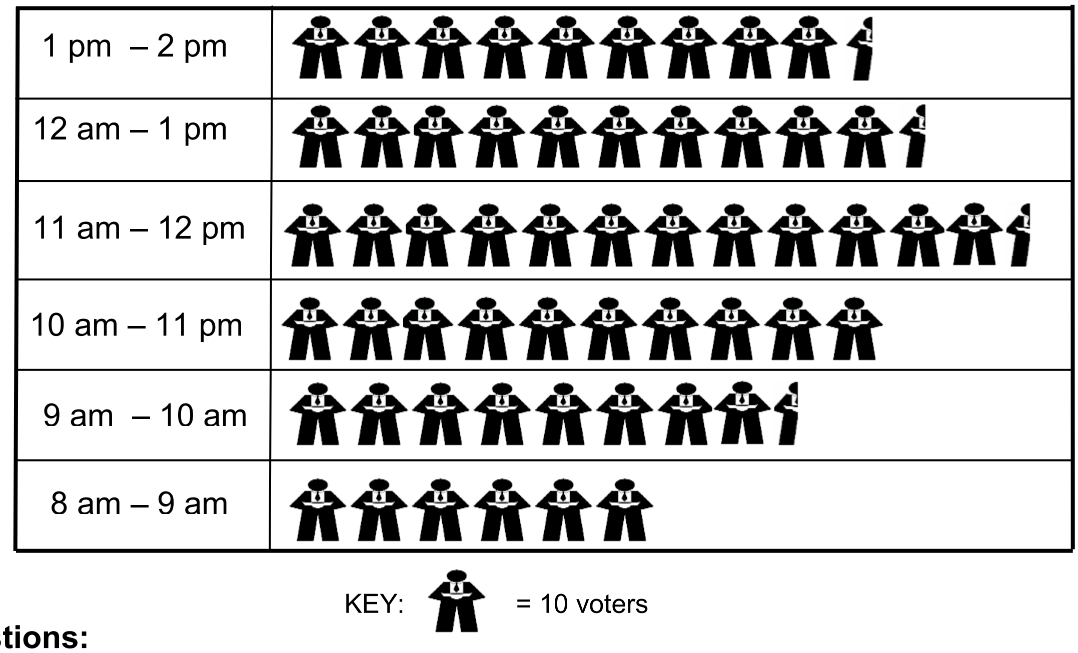
1. Refer to the picture graph below to answer the questions that follow.

NUMBER OF VOTERS ATTENDED
DURING THE FIRST 6 HOURS OF VOTING



Questions:

a) What is the title of the pictograph?

b) What does the graph represent?

c) What does half the man represent?

d) How many people attended during the first six hours of voting time?

d) At what time did the largest number of voters attend?

---

2. James is the treasurer of the Student Council in his school. When he was asked to report on the fund-raising activities of the council, he reviewed the council"s record books and made the following report:

2004 - 2005	-	Food Bazaar, K2,750
2005 - 2006	-	Rummage Sale, K4 000
2006 - 2007	-	Raffle, K 2000
2007 - 2008	-	Bingo Socials, K3 000
2008 – 2009	-	Film Showing, K3,750

Make a pictograph using the data presented by James in his report to the council.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1.

Lesson 2: Bar Graphs

In Lesson 1, you described pictographs and learned how to draw them.

In this lesson, you will:

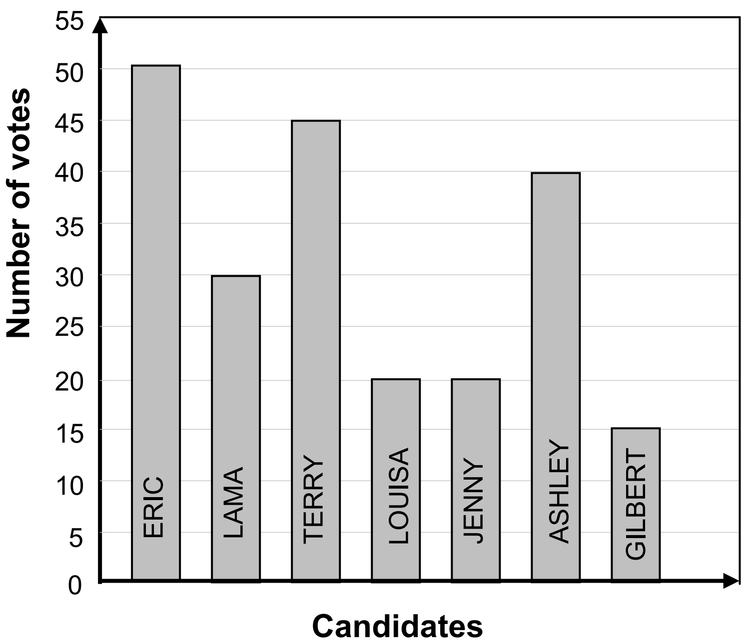
- describe a bar graph
- differentiate between types of bar graphs
- interpret information from bar graphs.

One of the most common types of graphs widely used in presenting data is the bar graph.

Let us show the election results in Table 1.1 on page 11.

Eric	- 50 votes
Lama	- 30 votes
Terry	- 45 votes
Louisa	- 20 votes
Jenny	- 20 votes
Ashley	- 40 votes
Gilbert	- 15 votes

FIGURE 2.1 **OSC LTD DIRECTORSHIP ELECTION RESULTS**



This type of graph is called a **bar graph.**

A bar graph is a graphical representation of data in which the frequencies are represented by a set of parallel bars of equal width and with lengths that were proportional to the frequencies.

A bar graph is used to compare things such as the votes for each candidate, classroom attendance, income and expense, marital status, scores of students in a test and so on.

Bar graphs present the statistical data in the form of columns or bars. Because the bar is drawn to exact amounts, it gives a clear indication of quantities. A number of bar graphs can provide a clear, visual impression of relative differences in quantities.

Bars graphs can be drawn vertically or horizontally. When the bars are drawn vertically it is called **a vertical bar graph or column graph.**

The first graph shown in the previous page (FIGURE 2.1) is called a vertical bar graph.

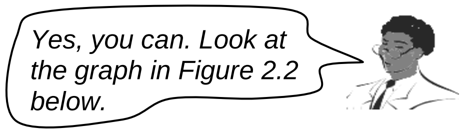
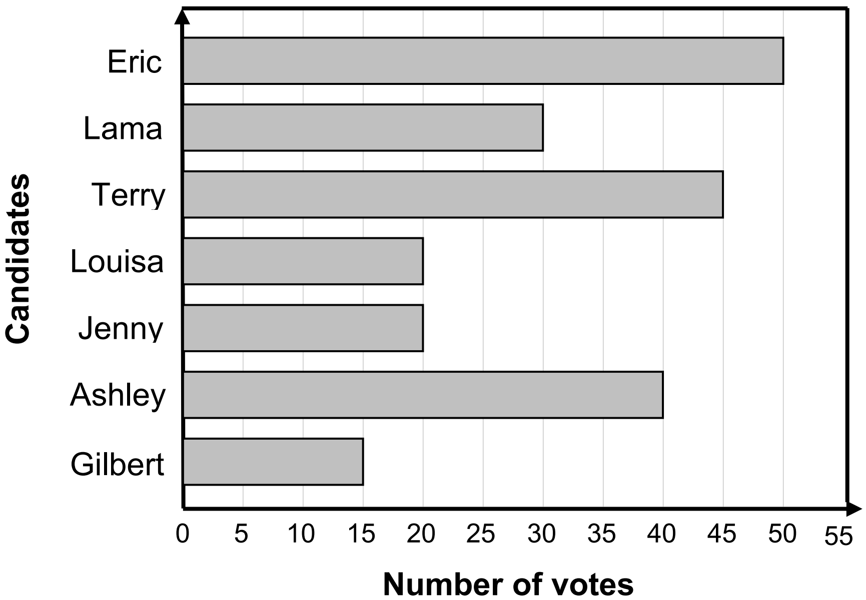


FIGURE 2.2

OSC LTD DIRECTORSHIP ELECTION RESULTS

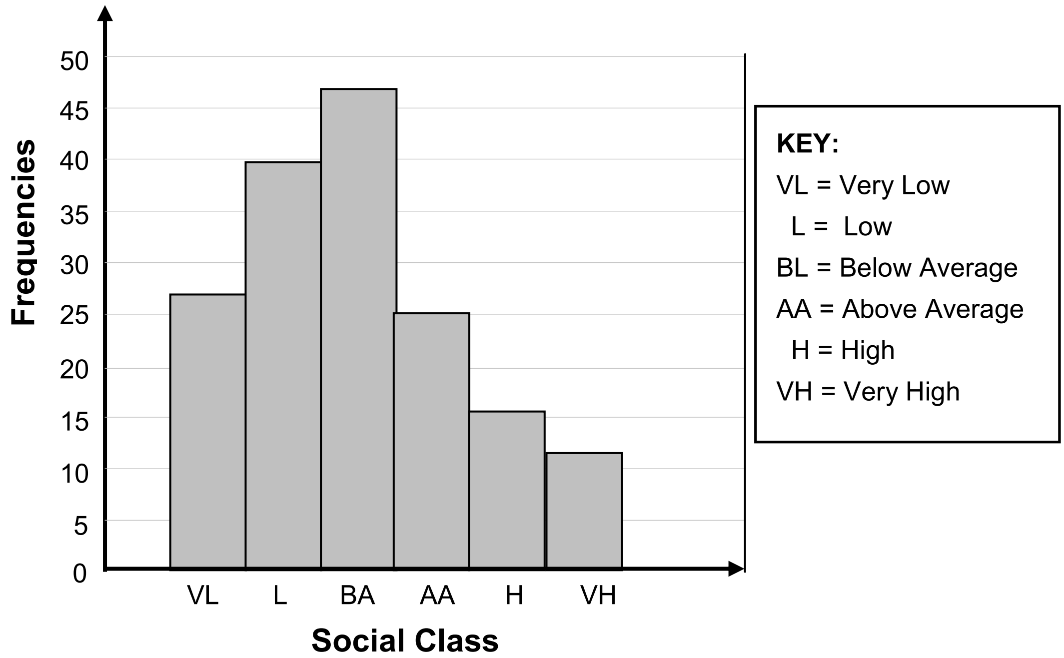


The graph is called a **horizontal bar graph**. The bars are drawn horizontally.

Note that in the bar graphs, the bars are separated from each other. This is because the categories are not continuous.
The bars should be drawn connected or joined in the case of continuous data to emphasize the degree of differences as shown in the next example. (Figure 2.3)

FIGURE 2.3

SOCIAL CLASS DISTRIBUTION OF GR 12 STUDENTS



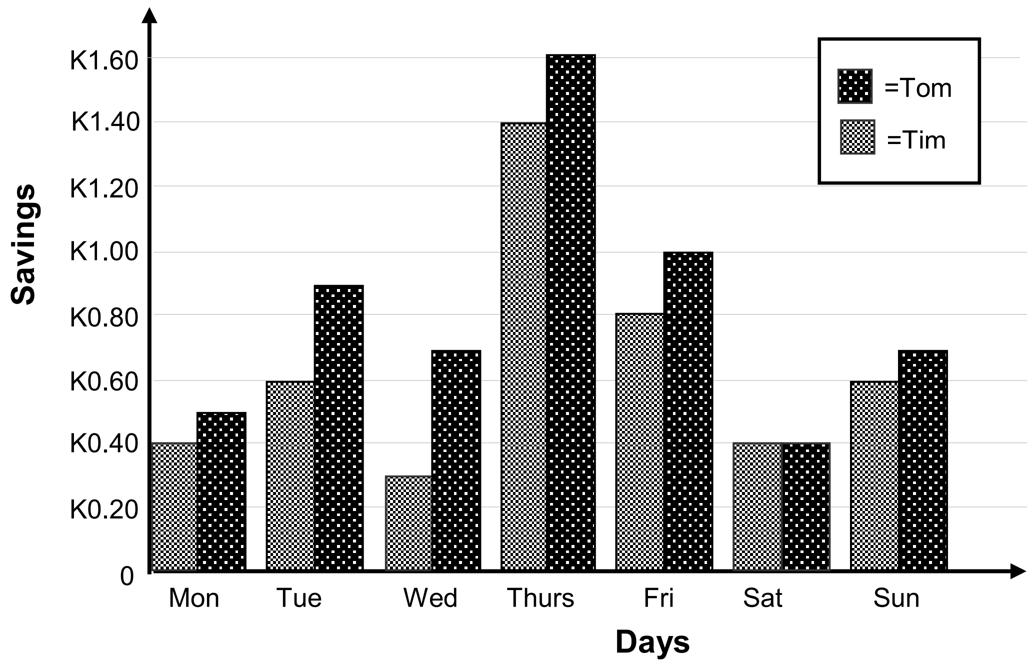
The figure shows a bar graph of the social class distribution of Gr 12 students. The bars are plotted in ascending order from the left to the right on the base line (horizontal axis). The bars represent from very low to very high social classes.

Now let us study the following graph in Figure 2.4.

The twin brothers Tim and Tom both study at Gordon‟s Secondary School. They both take a PMV to school. Their father gives each of them an allowance of K8.

FIGURE 2.4

TOM'S AND TIM'S SAVINGS IN A WEEK



The graph we have in FIGURE 2.4 is an example of a **double bar or dual bar graph**. It is a special type of graph which is used to compare data for two related items for the same period.

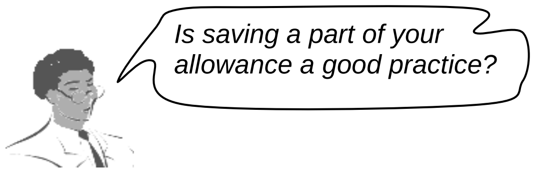
Now using the graph in Figure 2.4, answer the following?

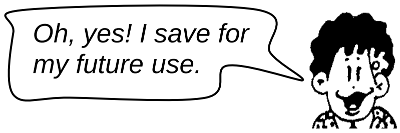
a. What is the graph about?

b. At a glance, can you tell who is thriftier?

c. What is the total savings for each of the two boys in a week?

d. How many percent of his allowance does Tom save in a week? How about Tim?

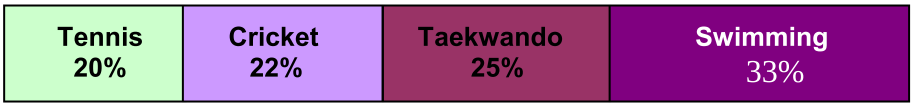




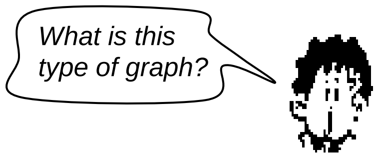
Now study Figure 2.5 below.

FIGURE 2.5

SPORTS for 210 YEAR 8 STUDENTS:



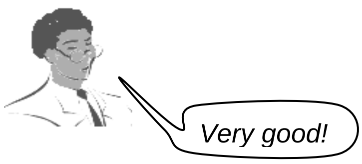
We call the graph a **divided bar graph.**

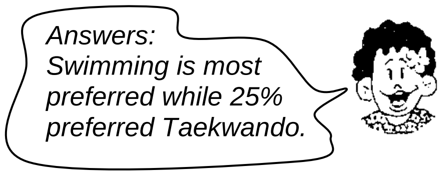


A divided bar graph is a type of graph where a bar or a rectangle is divided to show the make up of the data.

The graph shows the details concerning the sports preferences of 210 Gr 8 Students in POMIS.

Can you tell which sport is most preferred by POMIS students?
How many percent prefer Taekwondo?



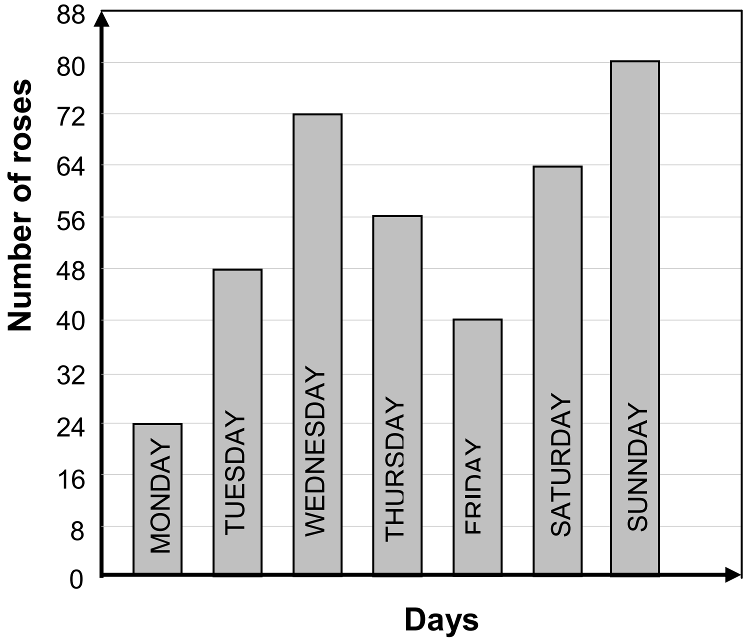


NOW DO PRACTICE EXERCISE 2

Practice Exercise 2

A. **Study the graph below and answer Questions 1 to 6.**

ROSES GATHERED BY MELAY DURING A WEEK



Questions

1. What kind of graph is this?

   ____

2. What does it compare?

   ____

3. Why are spaces provided between the bars?

   ____

4. On what day did Melay gather the most number of roses? How many roses did she gather?

   ____

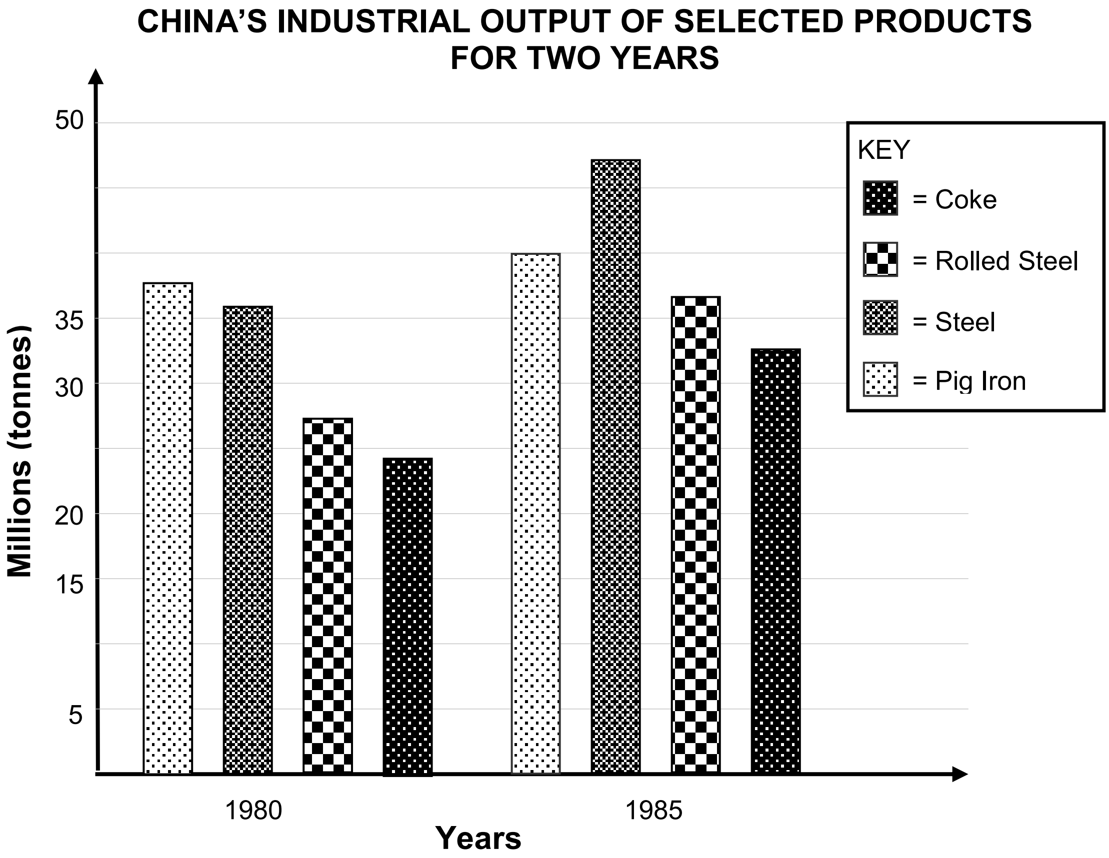
5. On what day did she gather the least number of roses? How many roses did she gather on this day?

   ____

6. How many roses did she gather during the week?

   ____

B. Study the graph below and answer the following questions.



Questions

1. Estimate the 1985 output of the following products.

   a. pig iron

   b. steel

   c. rolled steel

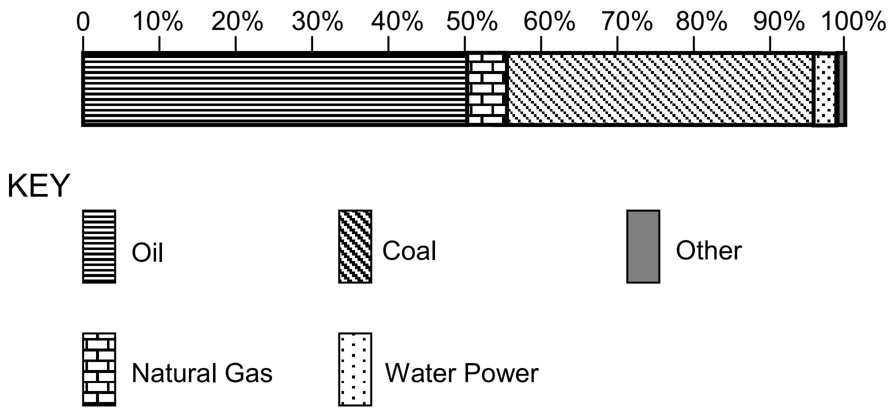
   d. coke

2. What unit of measurement has been used on this graph?

3. Which product has the greatest output in 1985?

C. Study the figure below.

1973 SOURCES OF ENERGY IN AUSTRALIA



Questions

1. What kind of graph is this?

2. What was the greatest source of energy in 1973?

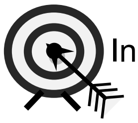
3. What percentage of energy came from oil?

4. List the sources of energy in order of size, from largest to smallest.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1

Lesson 3: Drawing Bar Graphs

In Lesson 2, you described bar graphs and differentiated between some commonly used types of bar graphs.

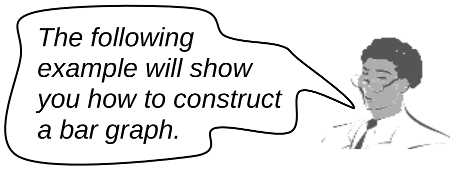


In this lesson, you will:

- identify parts of a bar graph
- draw bar graphs using given information
- solve problems involving bar graphs.

As we have learnt, bar graphs present statistical data in the form of columns or bars. Because the bar is drawn to exact amounts, it gives a clear indication of quantities.

In constructing bar graphs, **the horizontal base line or the x-axis** represents the categories and **the vertical line or the y-axis** represents the frequencies for each category. The **bars** that you draw constitute the frequencies for the range of each value. The shorter the bar, the smaller the frequency endorsement is. Remember that the bars should be of the same width. The spaces between the bars should be of the same width, but not necessarily the same as the width of the bars.



Example 1

Make a vertical bar graph of the data in Table 3.1 showing the eggs collected by Ramiro during a Week.

Table 3.1

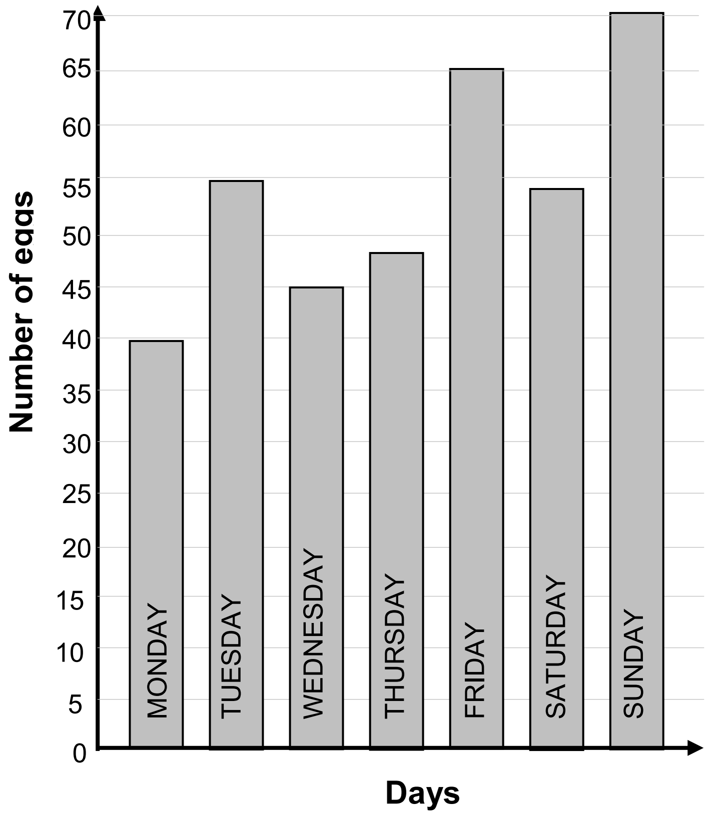
EGGS COLLECTED BY RAM DURING A WEEK	
Days	**Number of Eggs**
Monday	**40**
Tuesday	**55**
Wednesday	**45**
Thursday	**48**
Friday	**65**
Saturday	**54**
Sunday	**70**

Steps in constructing a bar graph

1. Draw the horizontal axis and decide upon the scale. The horizontal axis represents the independent variable – often expressed in time units.
2. Draw the vertical axis and decide upon the scale. Usually the scale will start with zero (0).
3. Decide upon the width and spacing of the bars. These should be uniform with the bars not being too thin or thick.
4. Draw the bars vertically or horizontally.
5. Label the parts.
6. Decide on a title for your graph.

Your graph must be similar to the graph below.

EGGS COLLECTED BY RAMIRO DURING A WEEK



Look at the figure and answer the following questions?

Understanding the graph

1. Estimate the total number of eggs collected by Ram during the week.
2. On what day did Ram gather the most number of eggs? How many did he gather on this day?
3. On what day did he gather the least number of eggs? How many eggs did he gather on this day?

Answers:	**1) 377 eggs**	**2) Sunday**	**3) Monday**

Now let us look at the steps in constructing a divided bar graph.

Let us go back to the details concerning the sports preferences of 210 Gr 8 Students on Figure 2.5 on page 17.

Sport chosen	Tennis	Cricket	Taekwando	Swimming	Total Students
Number of students	42	46	52	70	210

Steps:

1. Choose a title.

   We could use "**PREFERRED SPORT of 210 YEAR 8 POMIS STUDENTS**"

2. Choose an appropriate length for the bar.

   Usually the number in each category is converted to a percentage of the whole and a bar of length 12 cm is used.

   This allows each millimetre to represents 1% of students.

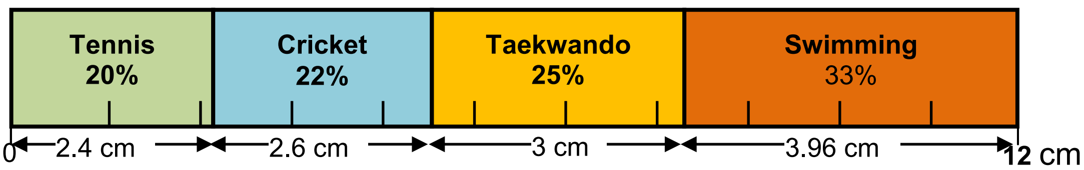
   Tennis = $\frac{42}{210}$ = 20% Cricket = $\frac{46}{210}$ = 22%

   Taekwando = $\frac{52}{210}$ = 25% Swimming = $\frac{70}{210}$ = 33%

3. Draw the graph.

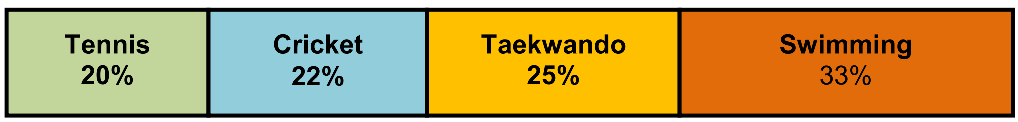
   We will make the length 120 mm (12 cm) so that 1% = 1 mm.

PREFERRED SPORT of 210 YEAR 8 POMIS STUDENT

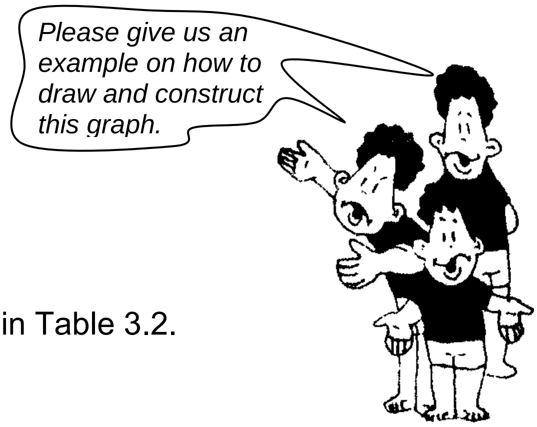


In a divided bar graph the marks to show units of measurements are not usually placed on the graph instead percentages are often shown, as in the case below.

PREFERRED SPORT of 210 YEAR 8 POMIS STUDENT



There will be times when you want to group certain items for the purpose of comparison. One way is to draw a column bar graph or a group bar graph.



Here is the example.

Example 2

Draw the bar graph for the following data in Table 3.2.

Table 3.2

URBANIZATION OF SELECTED COUNTRIES
1965-1908 AND 1980-1987

	Urban Percentage Of Total Population	
Country	**1965 – 1980**	**1980 – 1987**
China	18	38
India	19	27
Indonesia	16	27
Thailand	13	21
Australia	86	86
Japan	67	77

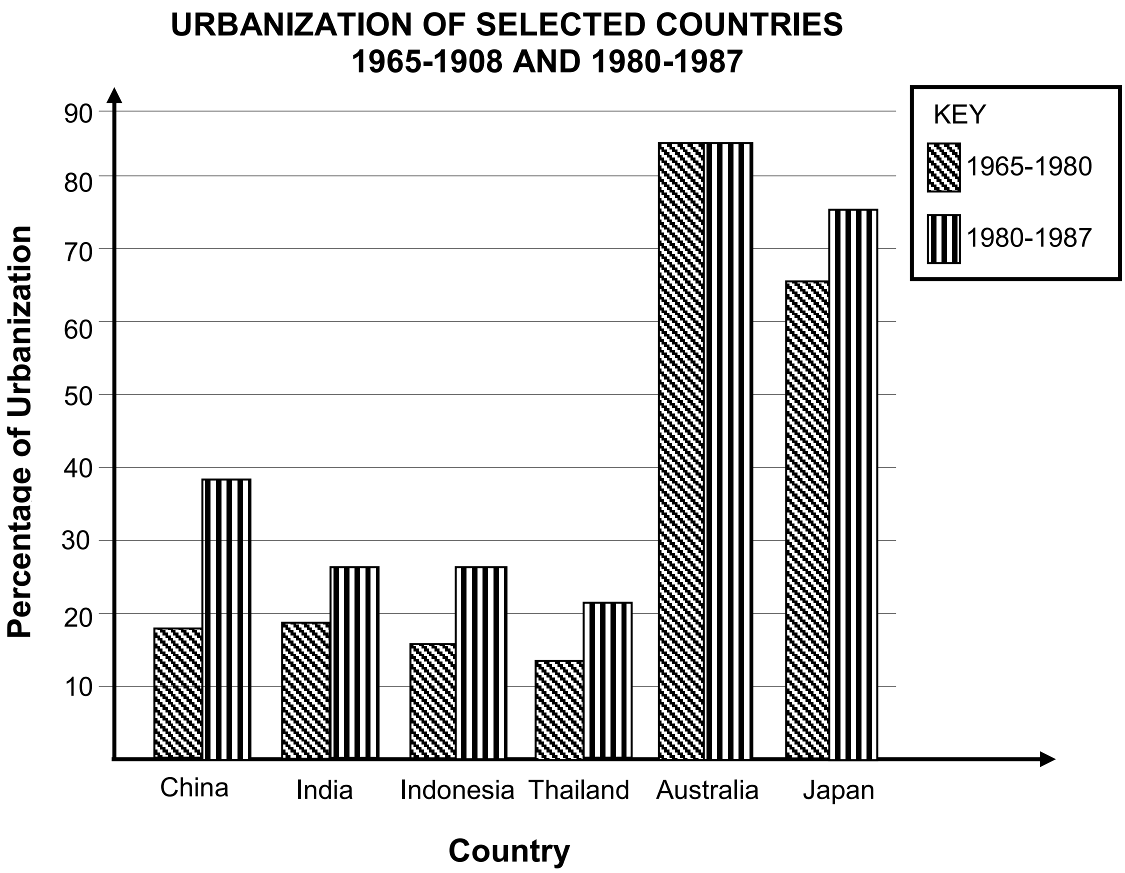
To draw the column or group bar graph for the data, a bar graph for the components for each year has to be drawn.

For example, a bar graph for China in 1965 - 1980 is drawn first, and then one for 1980- 1987 is drawn. The two bars are grouped together. These bars then clearly show relative quantities for each year.

The steps for drawing group or column bar graphs are similar to those used when we draw a simple bar graph, except that one bar graph is drawn for each component.

The component parts that have the same characteristics, such as the same time period, are grouped together.

When you finish drawing your graph it must be similar to the graph on the next page.



Understanding the graph

1. Which country had the greatest increase of urbanization? By how much?

Answer: China had the greatest increase of urbanization by 20%.

NOW DO PRACTICE EXERCISE 3.

Practice Exercise 3

1. Make a vertical bar graph to show the data in the table below about the growth rates in selected European Countries in 1981. Draw your graph on the box provided below.

GROWTH RATES IN SELECTED EUROPEAN COUNTRIES IN 1981	
Country	**Rates per hundred**
Romania	0.9
Spain	0.8
France	0.4
Finland	0.4
Switzerland	0.3
Italy	0.2

Draw your graph here.

2. Make a column graph to display the following data on the space provided in the box below.

HAZEL'S AND ANNE'S SAVINGS IN A YEAR

Month	Hazel	Anne
January	K32	K40
February	K30	35
March	20	25
April	18	24
May	40	40
June	28	35
July	54	60
August	35	45
September	35	40
October	45	55
November	50	55
December	75	85

Draw your graph here.

2. Make a divided bar graph on the box below to display the following information.

CHOICE OF ICE CREAM FLAVOUR

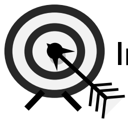
Flavour	Vanilla	Chocolate	Strawberry	Mango	Others
Number of children	40	60	80	36	24

Draw your graph here.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1.

Lesson 4: Line Graphs

In Lesson 3, you identified and drew the different types of commonly used bar graphs. You also solved problems related to bar graphs.



In this lesson, you will:

- describe line graphs
- identify commonly used types of line graphs
- interpret data presented in a line graph

You learnt the meaning of Line graphs in your Grade 7 Mathematics.

Here is the description again in case you forgot.

A line graph is used to show changes and relationships between two sets of quantities or data. One is represented by horizontal lines on the graph and the other is represented by vertical lines. The line segments connecting the intersections of the representations is the graph of the relationship between the two sets of quantities.

Lines are used to plot the rise and fall of quantities. As the line rises and falls, it clearly shows change, so it is easy to read the amount of change from these kinds of graphs.

Some of the data which can be represented by line graphs include: temperature, population, crop production, mineral production, exports and imports, average annual rainfall, employment, exchange rates, weights, heights and distances.

There are different types of line graphs. In this lesson we will examine only simple line graphs, conversion graphs, step graphs and travel graphs.

Let us study an example of each type of line graph.

Look at the Figure 4.1

ONEA'S JOURNEY HOME

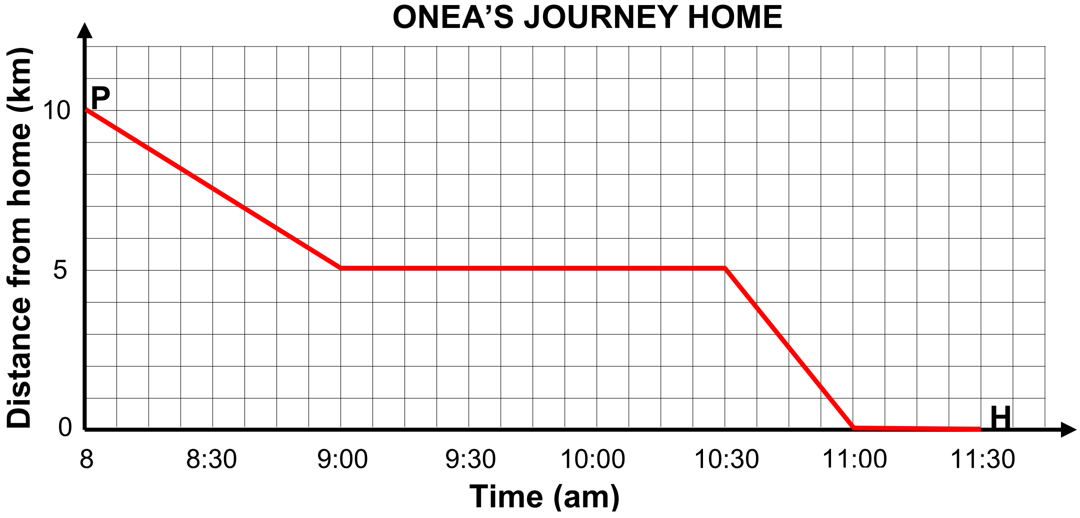


Figure 4.1 is an example of a **travel graph**.

The graph shows that Onea travelled from a place (P) to his home. His distance from the place as time passed by is given by the graph. During his trip, he travelled 5 km before he stopped at 9:00 am. He rested for one and a half hours then travelled slowly towards his home where he arrived at 11:00 am and rested.

Understanding the graph we have the following notes and observations.

- The distance from home is shown on the vertical axis, so when Onea reaches home (H) he is 10 km from the place (P)
- When the graph slopes down as time passes, Onea is travelling away from P.
- When the line is horizontal he has stopped.
- He is 10 km from P and reaches home at 11:00 pm and rested till 11:30 am.

A travel graph is a line graph that relates distance and time to show progress on a trip.

Time is shown on the horizontal axis and distance from a particular place is shown on the vertical axis.

The steeper the graph, the greater the speed is.

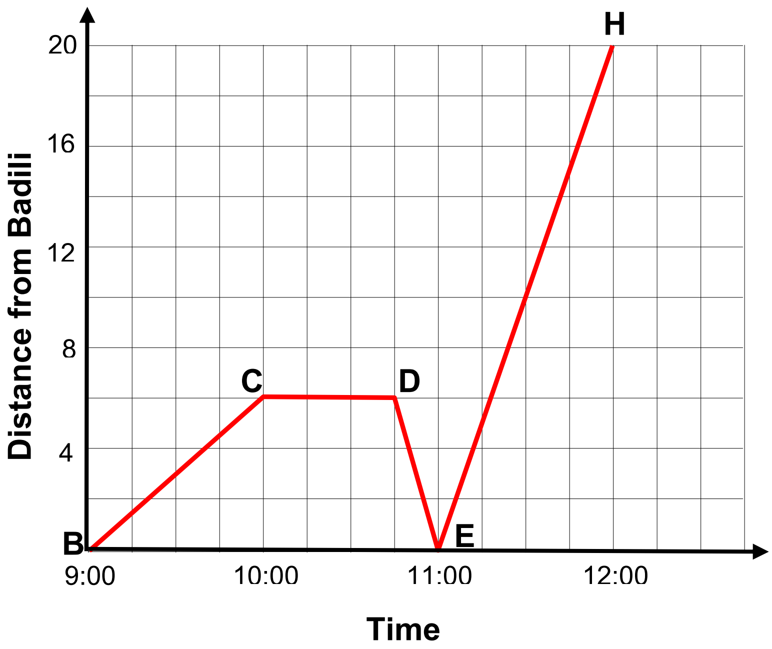
If the graph is horizontal, the person has stopped. Speed is zero, as it is assumed that travel is along a straight road.

Example 1

Study the following information concerning Figure 4.2 below.

Sebastian travelled from Badili (**B**) to his home (**H**) along the 2 Mile Road. His distance from Badili as time passed is given by the graph. During his trip, Sebastian stopped for a while, then returned to Badili, then travelled to his home where he arrived at 12 noon.

SEBASTIAN'S JOURNEY



Understanding the graph.

a) Sebastian travelled 5 km from Badili (**BC**) before he stopped. He had travelled slowly (5 km/h) stopping at 10 am.

b) He waited for 3 quarters of an hour (**CD**) and then returned to Badili more quickly than he had come away (**DE**)

c) He then, very quickly (20 km in one hour) went home (**EH**) arriving at 12 noon.

Explaining the graph.

- The distance from Badili is shown on the vertical axis, so when Sebastian reaches home (H) he is 20 km from Badili.
- When the graph slopes up as time passes, Sebastian is travelling away from Badili. When the graph slopes down, Sebastian is moving towards Badili.
- The steeper the line the greater is his speed.
- When the line is horizontal he has stopped.
- He is 20 km from Badili when he reaches home at 12:00 noon.

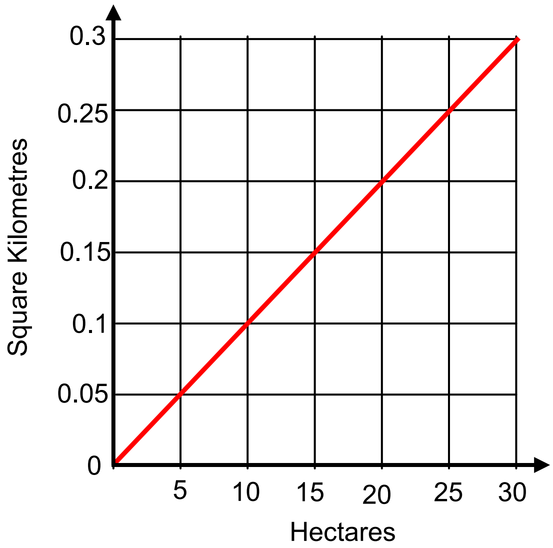
Example 2

The line graph on the right shows conversion of hectares to square kilometres.

Looking at the line graph we see that 30 hectares (ha) = 0.3 square kilometres (km^2) and that 0 hectares = 0 square kilometres.

We need to go up to 30 on the hectares axis and 0.3 on the square kilometres axis.

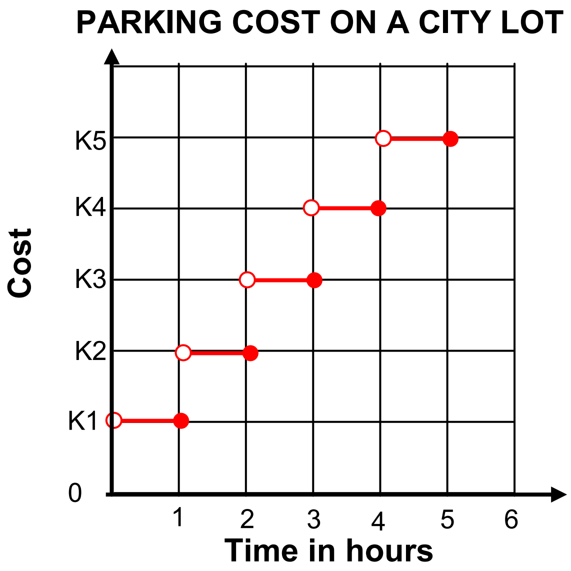
HECTARES TO SQUARE KILOMETRES CONVERSION



This graph is called a **conversion graph.**

A conversion graph is a line graph that allows us to change from one unit to another. **For example: Conversion of metric units** **Conversion from Celsius to Fahrenheit** **Conversion of currency**

Now let us look at the graph below.



This kind of line graph is called a **step graph**.

> **A step graph is a line graph which is made up of several horizontal intervals or steps to show the information. The end of one horizontal step and the start of the next one are in the same vertical line. Each horizontal step has an open circle (○) and a closed circle or dot (●) at its end. An open circle excludes that point from the step. A closed circle or dot includes that point in the step.**

Thus, if we look at the graph above, when "time" equals "3", the "cost" is, K3 not K4.

Note: When a gap or jump appears in a graph, we say that the graph is discontinuous.

Understanding the graph above, let us answer the following questions.

1. What unit is used on the horizontal axis?

2. How much should it cost to park for
   a) 30 minutes?
   b) 3 hours?
   c) 4 hours 30 minutes?

> **Answers;**
> 1. hours
> 2. a) K0.50
>   b) K3
>   c) K4.50

NOW DO PRACTICE EXERCISE 4

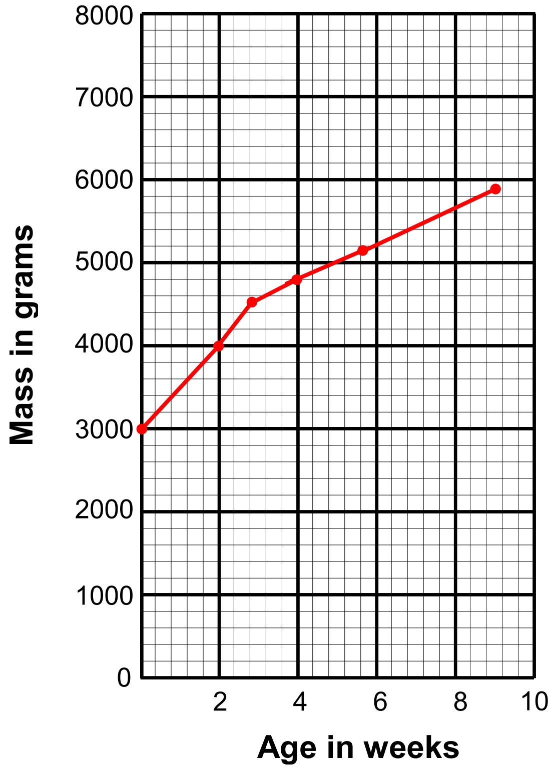
Practice Exercise 4

1. Fill in the blanks.

A line graph is used to show __________ and __________ between two sets of quantities or data. One is represented by __________ on the graph and the other is represented by a ___________. The line segments connecting the intersections of the representations is the __________ of the relationship between the two sets of quantities.

2. Refer to the line graph below to answer the questions that follow it.

ANTON'S MASS IN HIS FIRST 9 WEEKS



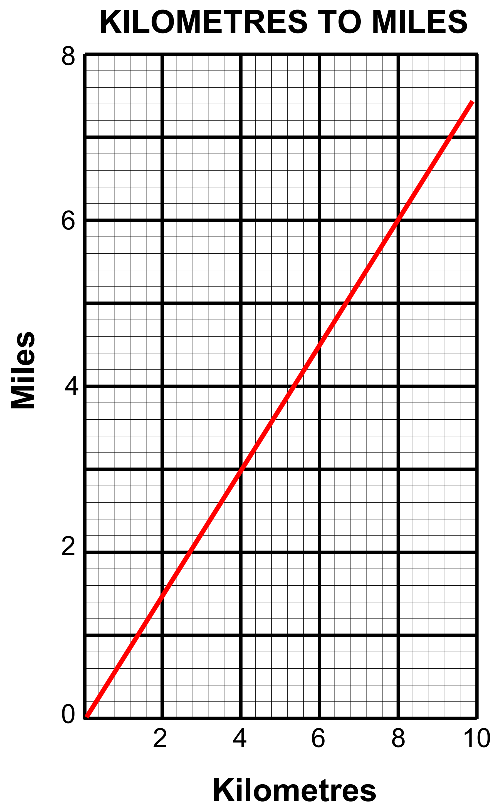
Questions:

a) What was Anton"s mass at the end of 9 weeks?

b) At what age did Anton have a mass of 4000 grams?

c) Within which week did Anton"s mass first pass 5000 grams?

3. Refer to the graph below and answer the questions that follow.



Questions:

a) How many miles are in 8 kilometres?

b) How many kilometres are in 5 miles

c) To the nearest kilometres, how many kilometres are in:

  i. 2 miles?

  ii. 7.5 miles?

  iii. 5.5 miles?

d) To the nearest mile, how many miles are in:

  i. 2 km?

  ii. 5 km?

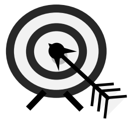
  iii. 11 km?

e) How many kilometres would be in 10 miles?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1.

Lesson 5: Drawing Line Graphs

In lesson 4, you identified and defined the different types of commonly used line graphs. You also learnt to interpret data presented in a line graph.

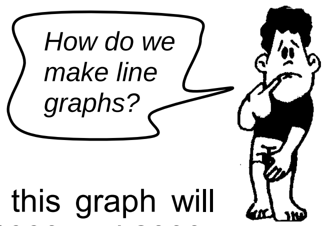


In this lesson, you will:

- identify parts of a line graph
- draw line graphs using given information
- solve problems related to line graphs

As we learnt earlier, lines are used to plot the changes and relationships between two sets of quantities. One is represented by horizontal lines on the graph and the other is represented by vertical lines. The line segments connecting the intersections of the representations is the graph of the relationship between the two sets of quantities.

Now we will look at how to construct line graphs.



Use the data in Table 5.1 to construct your own line graph, by following the steps detailed as follows. When complete, this graph will show actual changes in the population of country X between 1930 and 2000.

Table 5.1
POPULATION OF COUNTRY X FROM 1930- 2000

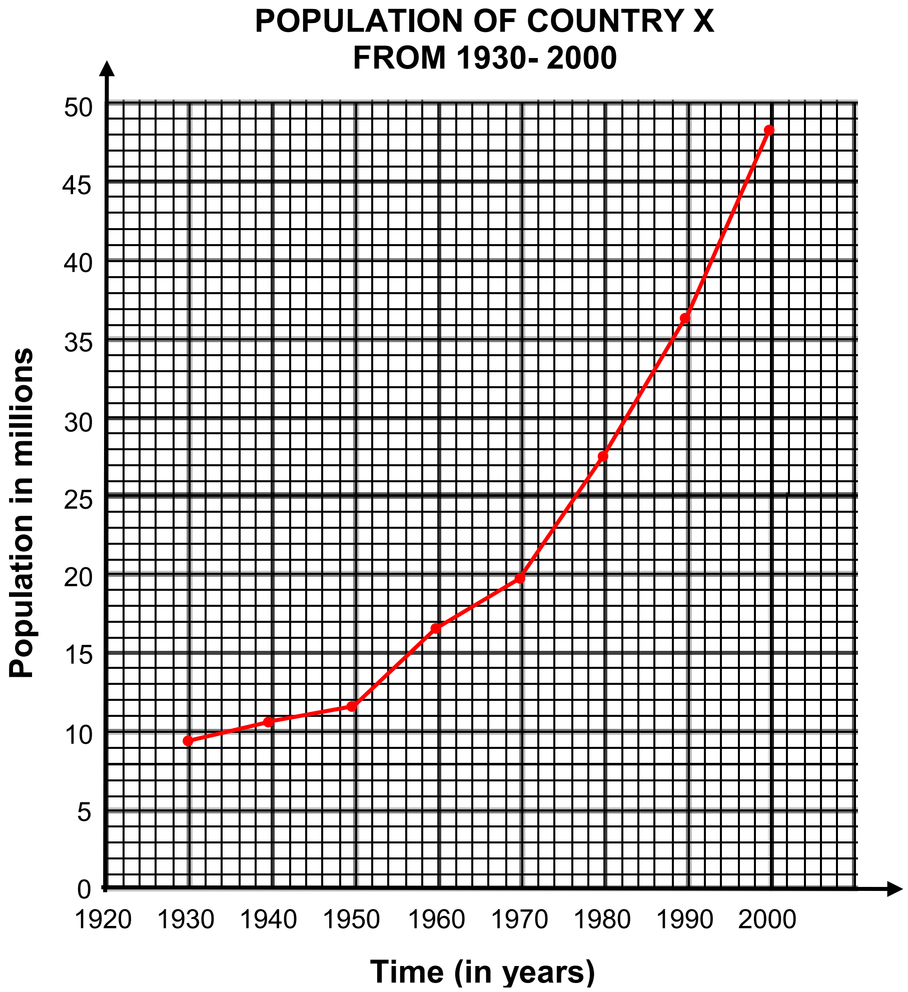
Year	Population (in millions)
1930	8.2
1940	10.4
1950	13.1
1960	16.5
1970	19.9
1980	27.1
1990	36.7
2000	48.1

Steps:

1. Use a squared grid paper.
2. Select the data to plot on the horizontal axis. This is usually the independent variable. In many graphs, this is the time axis.
3. Select the data to plot on the vertical axis. This is usually the dependent variable, that is, the data that rises or falls over time.
4. Draw the horizontal and vertical axes. Make sure the scales and their endpoints suit the data being plotted.
5. Label the axes. State what is being represented.

6. Plot the points.
7. Connect the points in the order that they are plotted.
8. Compose a title to your graph.

Your graph must be similar to the graph below.



Let us study the graph using the given data.

The horizontal axis is for the years. Since there are eight ten-year periods represented, eight equal spaces are provided on the horizontal axis. The vertical axis is used as the reference line for the population. Each space on the vertical axis represents 5 million in population. The bottom dot represents the 8.2 million for 1930, the next dot 10.4 million and so on.

It is meaningful to connect points with line segments because in this case, the points on the line segments give us the estimate of the population between years.

For example, the year 1945 is represented by the vertical line between 1940 and 1950. This line intersects the line graph. From the point of intersection, we go left and follow a horizontal line to the vertical axis where the population numbers are shown. The point is 10 and 15 or 12.5. An estimate of the population in 1945 is 12.5 million. Can you now give estimates of the population in 1995?

NOW DO PRACTICE EXERCISE 5

Practice Exercise 5

1. Show the following in a line graph.

   a. World Population growth, 1950 to 2010

Table 5.2

Year	Population (in Billions)
1950	2.2
1960	3.0
1970	3.7
1980	4.4
1990	4.1
2000	6.1
2010	8.2

2. A kilogram of rice costs K4.65. Complete the table below. Draw a line graph to show these data.

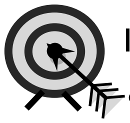
Table 5.3

Number of Kilograms	Price
I	K4.65
2	K9.30
3	___
4	___
5	___
6	___
7	___
8	___
9	___
10	___

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1.

Lesson 6: Pie Graphs or Circle Graphs

In Lesson 5, you identified the parts of a line graph and drew line graphs using information provided.



In this lesson, you will
- describe a pie or circle graph
- interpret information from pie or circle graphs.

There are times when line and bar graphs are not appropriate. So then we need to use another type of graph. In this section, we will study and explain one of them, the one we call a **pie graph** or a **circle graph**.

Very good! This time you will extend further your knowledge about them.

I remember, studying pie graphs or circle graphs in Grade 7

Again, let us discuss the meaning and use of a **pie graph** or a **circle graph.**

A circle graph or a pie graph is a type of graph which is used to show the relationship of a part to a whole quantity.

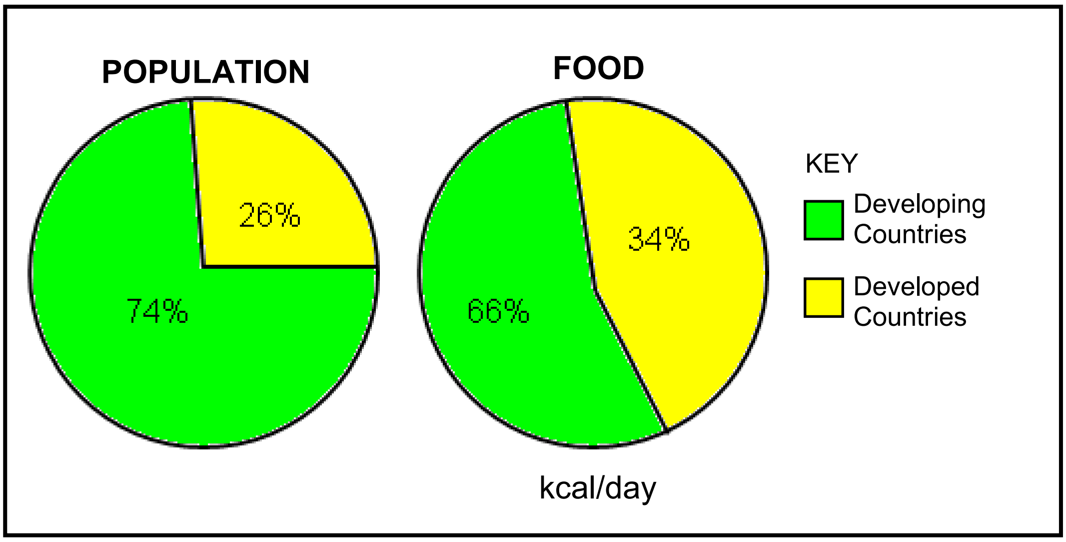
Some examples of these are relationships between items of expenses and savings, percentage distribution of populations, food consumption and the like.

When a circle is divided into sectors which, taken together, represent a whole, it is easy to compare the relative sizes of the components. Each sector of the graph looks like a slice of pie and represents a proportion of the total. A table of data can be converted to a pie graph for easy visual comparison and at the same time the graph is attractive yet it takes up little space.

Look at the following figure.

Figure 6.1

WORLD AVERAGE FOOD CONSUMPTION FOR 2000 - 2002

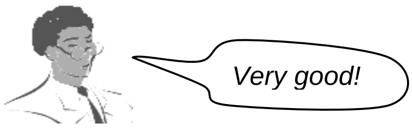


The graphs in Figure 6.1 are examples of simple circle graphs or pie graphs.

Let us analyse and understand the graph, by answering the following questions.

1. What percentage of the total world population is living in developed countries? in developing countries?

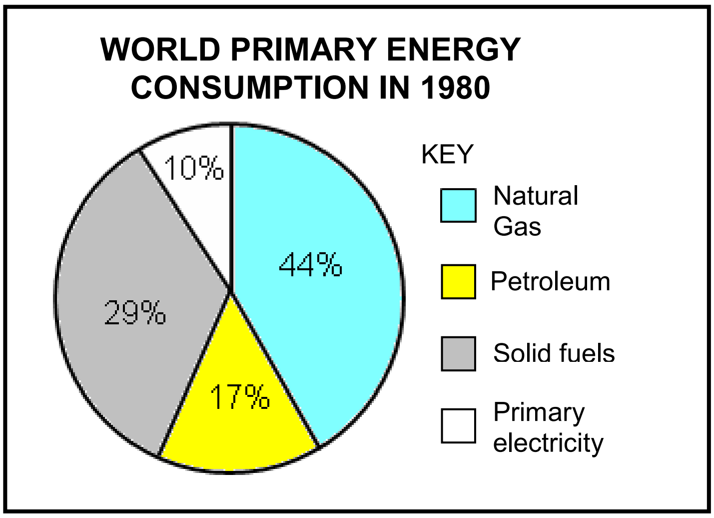
2. What percentage of total world food consumption occurs in developed countries? in developing countries?



Here is another example.

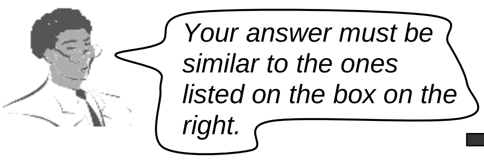
Let us study and understand the graph in Figure 6.2 below.

Figure 6.2



Looking at and understanding the graph, you see that the graph shows and illustrates information about the world primary energy consumption in 1980. Can you state the answers for the following questions based on your understanding?

1. Write the percentage of world energy consumption of petroleum in 1980?
2. What was the percentage of world energy consumption of primary electricity in 1980?
3. Rank the components that make up the world‟s primary energy consumption from most important to least important in 1980.



Answers:

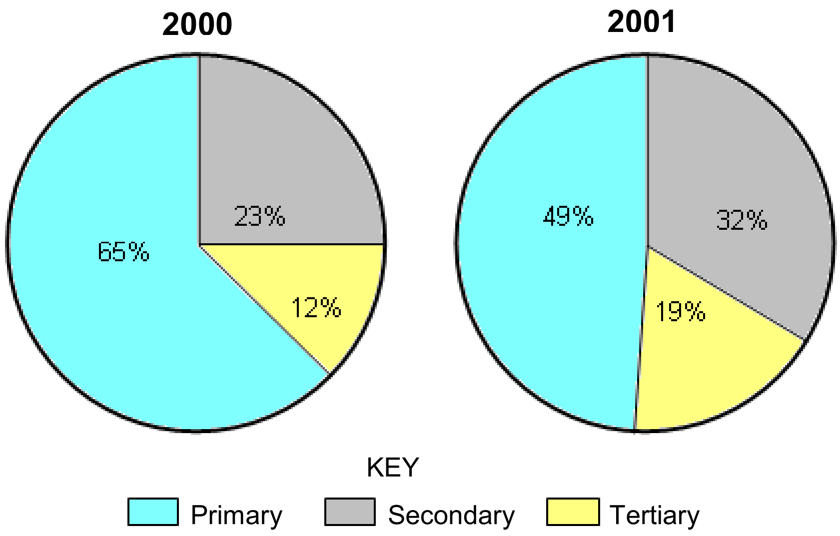
1. 17 %
2. 10%
3. Natural gas - rank 1
   Solid fuels - rank 2
   Petroleum - rank 3
   Primary electricity - rank 4

NOW DO PRACTICE EXERCISE 6

Practice Exercise 6

1. Refer to the figure below and answer the questions that follow.

SECTIONAL DISTRIBUTION OF THAI WORKFORCE, 2000 AND 2001



Questions:

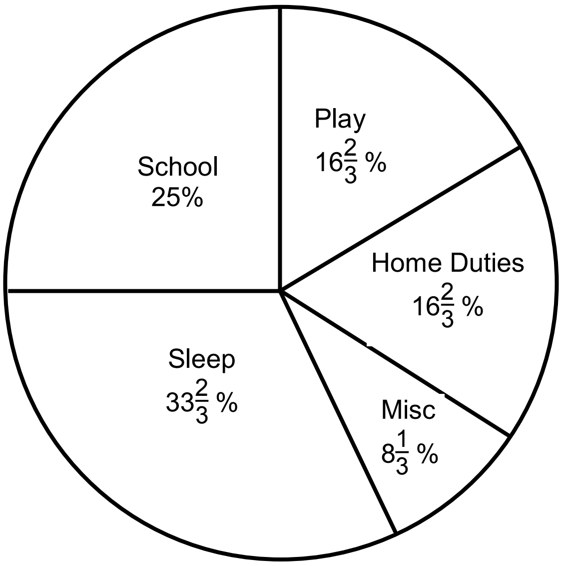
a. Which sector of the workforce was greatest in 2000?

b. What proportion of the Thai workforce was involved in tertiary industries in 2000?

c. What proportion of the Thai workforce is estimated will be involved in secondary industry by 2001? What change is this from 2000?

d. Identify the changes in the workforce of Thailand in three sectors of the economy between 2000 and 2001.

2. The graph below shows how a Grade 7 student spends a typical day.



Questions;

a. Find the total percent given in the graph.

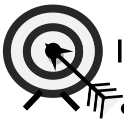
b. How many hours per day does the boy sleep? play?

c. What fraction of the day does the boy spend in school?

CORRECT YOUR WORK ANSWERS ARE AT THE END OF SUB-STRAND 1.

Lesson 7: Drawing Pie Graphs or Circle Graphs

In Lesson 6, you learnt to interpret and explain information from a given pie graph or circle graph by answering questions based on the graph.



In this lesson, you will:

- identify the steps in drawing or constructing a pie graph or circle graph
- draw or construct pie graphs
- solve problems related to pie graphs.

As we know, a circle graph or a pie graph as the name suggests, consists of a circular region divided into sectors that do not overlap, and each sector represents a part or a percentage of the whole being considered. The whole area of the circle represents the total value.

In this lesson, you will need to apply your skills in measuring and drawing angles using a protractor and a compass in order to construct a circle graph.

Let us use the information in the following example and the steps below to construct a circle graph or pie graph.

A study of car accidents shows that 55% of all accidents are caused by reckless driving, 25% by carelessness, 15% by faulty equipment, 5% by other causes. Make a circle graph to show the facts discovered in this study.

Steps:

1. Using your compass, draw a circle of whatever size you want, but note that very small circles are harder to work with.

2. The percentages in the example information need to be converted into degrees. Do this by multiplying each percentage by 360. This gives the angle of each segments of the circle which corresponds to these percentages.

   a. 55% x 360 = 0.55 x 360 = 198 degrees

   b. 25% x 360 = 0.25 x 360 = 90 degrees

   c. 15% x 360 = 0.15 x 360 = 54 degrees

   d. 5% x 360 = 0.05 x 360 = 18 degrees

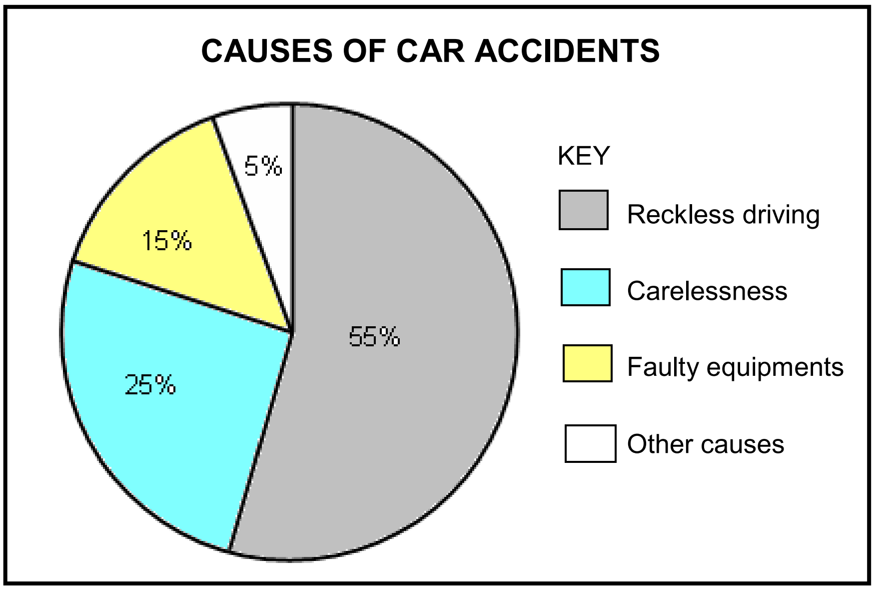
3. Use a protractor to draw angles whose measures are 198º, 90º, 54º and 18º at the centre of the circle you have drawn in step 1. Arrange the angles in ascending order, working in clockwise or counter clockwise direction.

4. Angles should be measured on a cumulative basis, e.g. after the first angle is plotted, the second angle is added to the first and the total plotted. Continue measuring in this cumulative way.

5. Write the percentages for each segment. You can then label each segment or shade the segments with different shades and include the key to identify them.

6. Make up a title for your graph.

After completing all the steps, your graph must be similar to the graph below.



Now, let us draw a circle graph to show the sports preferences of a group of Grade 8 students in St. Mary‟s Academy as shown below.

SPORT	No. Players
Soccer	20
Basketball	16
Volleyball	8
Swimming	4

Procedure:

1. Determine the measure of the angles at the centre of the circle.

   a. Add 20 + 16 + 8 + 4. You get 48.

   b. Find what part of the group prefers each kind of sport.

$$\frac{20}{48} = \frac{5}{12}\ ; \quad \frac{16}{48} = \frac{1}{3}\ ; \quad \frac{8}{48} = \frac{1}{6}\ ; \quad \frac{4}{48} = \frac{1}{12}$$

c. Determine how many degrees correspond to these fractional parts by multiplying each fraction by 360.

$\frac{5}{12} \times 360^\circ = 150^\circ$; $\frac{1}{3} \times 360^\circ = 120^\circ$

$\frac{1}{6} \times 360^\circ = 60^\circ$; $\frac{1}{12} \times 360^\circ = 30^\circ$

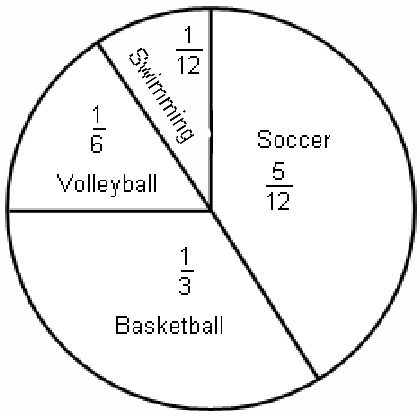
2. Draw a circle and with your protractor, draw angles at the centre of the circle whose measures are 150º, 120º, 60º and 30º. Extend the sides of these angles to the circle. You now have sectors of a circle.

3. Label your sectors. You can either label each sector, or shade the sectors differently and use a key to identify them.

4. Make up a title.

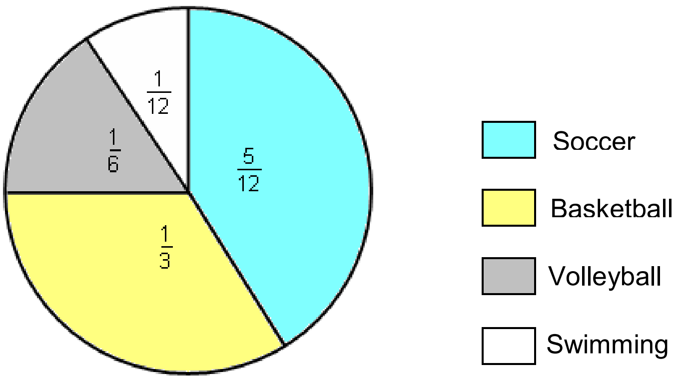
You now have a circle graph similar to the graph shown below.

CHOICE OF SPORTS BY ST. MARY'S ACADEMY GRADE 8 STUDENTS



or like this below.

CHOICE OF SPORTS BY ST. MARY'S ACADEMY GRADE 8 STUDENTS



NOW DO PRACTICE EXERCISE 7

Practice Exercise 7

1. Make a circle graph showing the details of the make –up of Koseteng Central School given below.

Infants	Primary	Secondary	Tertiary	Total
30	54	60	16	160

Draw your graph here.

2. Use a circle graph to present the national government expenditures for 1982 as recorded in the 1983 Country **A** statistical Yearbook and answer the questions that follow:

Economic services	46%
Education	10%
Housing, population and public health	8%
Social welfare	2%
National defence	10%
General public service	24%

Questions:

a) Which item has the largest expenditure?

b) Which item has the smallest expenditure?

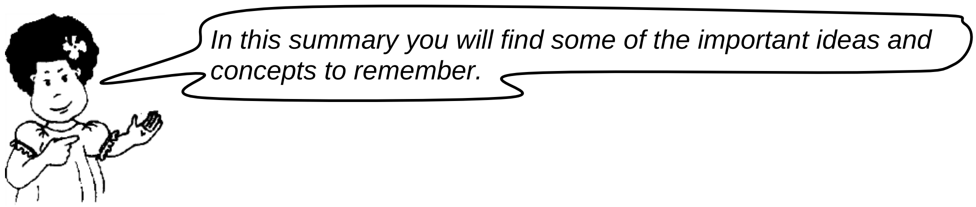
c) If the total expenditure in 1982 was K60 billion, how much was set aside for education? for national defence?

d) Approximately how many times that of housing, population and public health is the expenditure for general public services?

Draw your circle graph here.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 1.

SUB-STRAND 1: SUMMARY



- Pictograms or pictographs show information by using pictures as symbols to represents quantities.
- Bar graphs are graphical representations of data in which the frequencies are represented by a set of parallel bars or rectangles of equal widths and lengths proportional to the frequencies. A travel graph is a line graph that relates distance and time to show progress on a trip.
  Some commonly used types of line graphs are:
  - ❖ Double or Dual Bar Graphs – a bar graph that is used to compare data for two related items of the same period.
  - ❖ Divided Bar Graph – a bar graph where a bar or a rectangle is divided to show the make-up of the data.
- Line graphs are used to show changes and relationships between two sets of quantities. One is represented by horizontal lines on the graph and the other is represented by vertical lines. The line segments connecting the intersections of the representations is the graph of the relationship between the two sets of quantities.
  Some commonly used types of line graphs are;
  - ❖ Travel graph- a line graph that relates distance and time to show progress on a trip.
  - ❖ Conversion Graph – a line graph that allows us to change from one unit to another.
  - ❖ Step Graph – a line graph which is made up of several horizontal intervals or steps to show the information.
- Pie Graphs or Circle Graphs are types of graphs which are used to show the relationship of a part to a whole quantity.

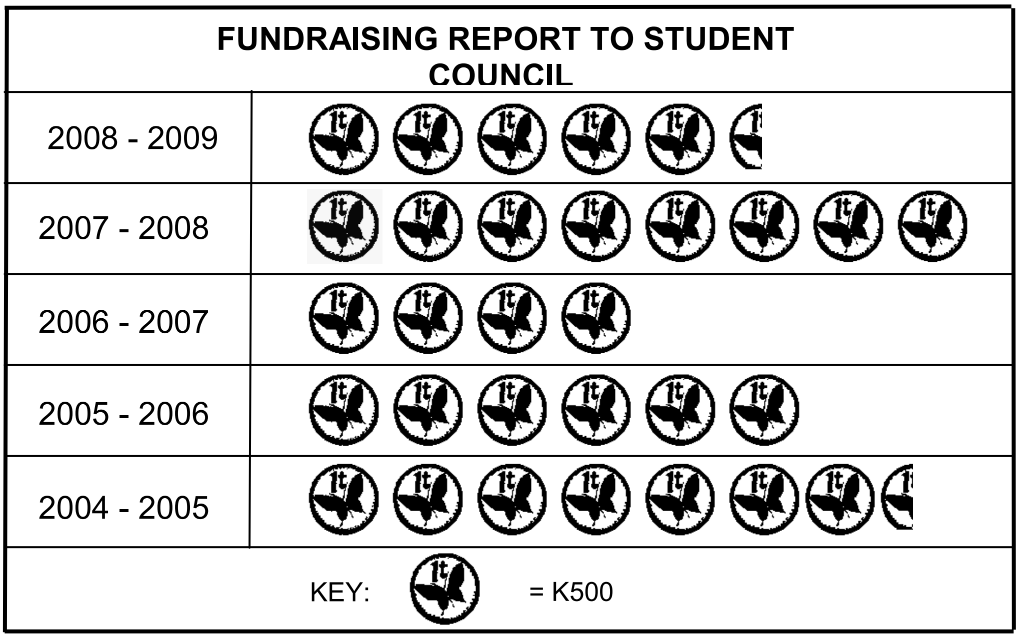
REVISE LESSONS 1-7 THEN DO SUB-STRAND TEST 1 IN ASSIGNMENT 1.

ANSWERS TO PRACTICE EXERCISES 1- 7

Practice Exercise 1

1. a) Number of Voters During the First 6 Hours of Voting
   b) It represents the total number of people who attended during the first 6 hours of voting.
   c) A half man represents 5 voters
   d) 570 voters
   e) 11 am to 12 pm

2.



Practice Exercise 2

A. 1. Bar Graph
   2. Number of roses collected per day in a week
   3. Because the categories are not continuous.
   4. Sunday; 80 roses
   5. Monday; 24 roses
. 6. 384 roses

B. 1. (a) 40 million tonnes
      (b) 47 million tonnes
      (c) 37 million tonnes
      (d) 33 million tonnes

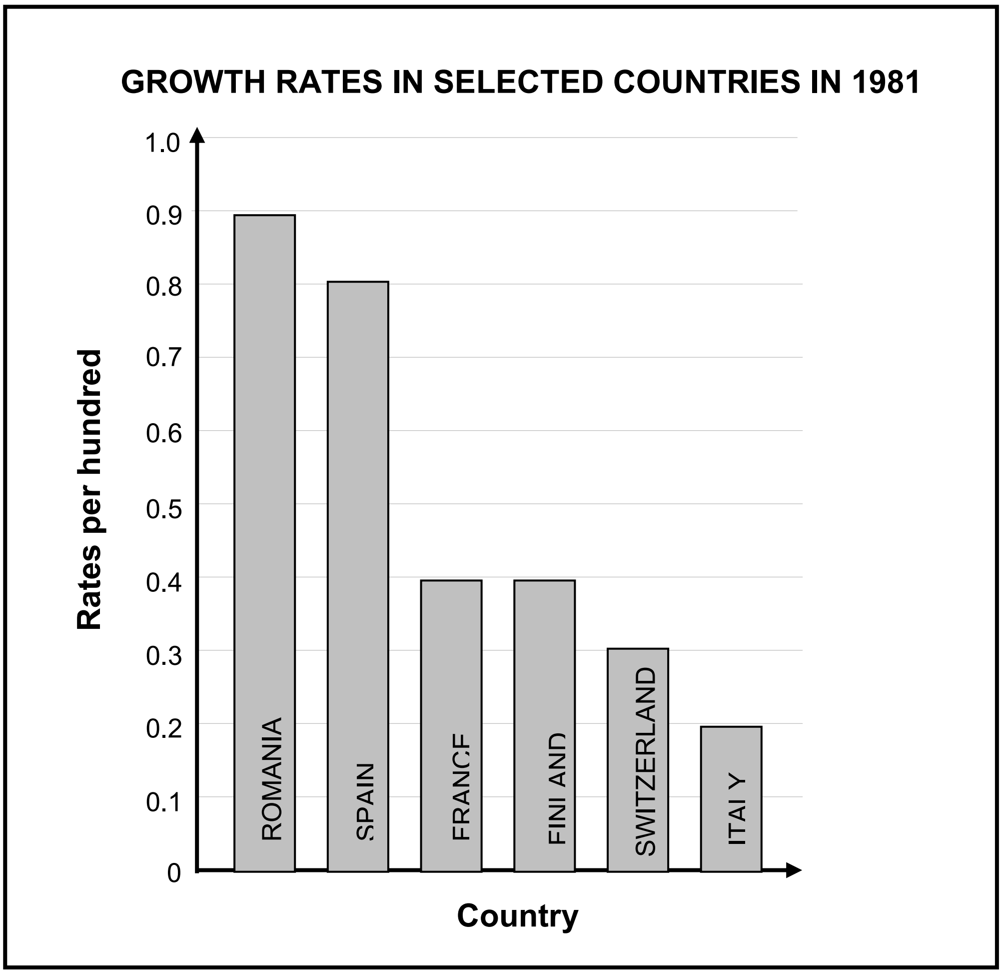
2. million tonnes per year

3. steel

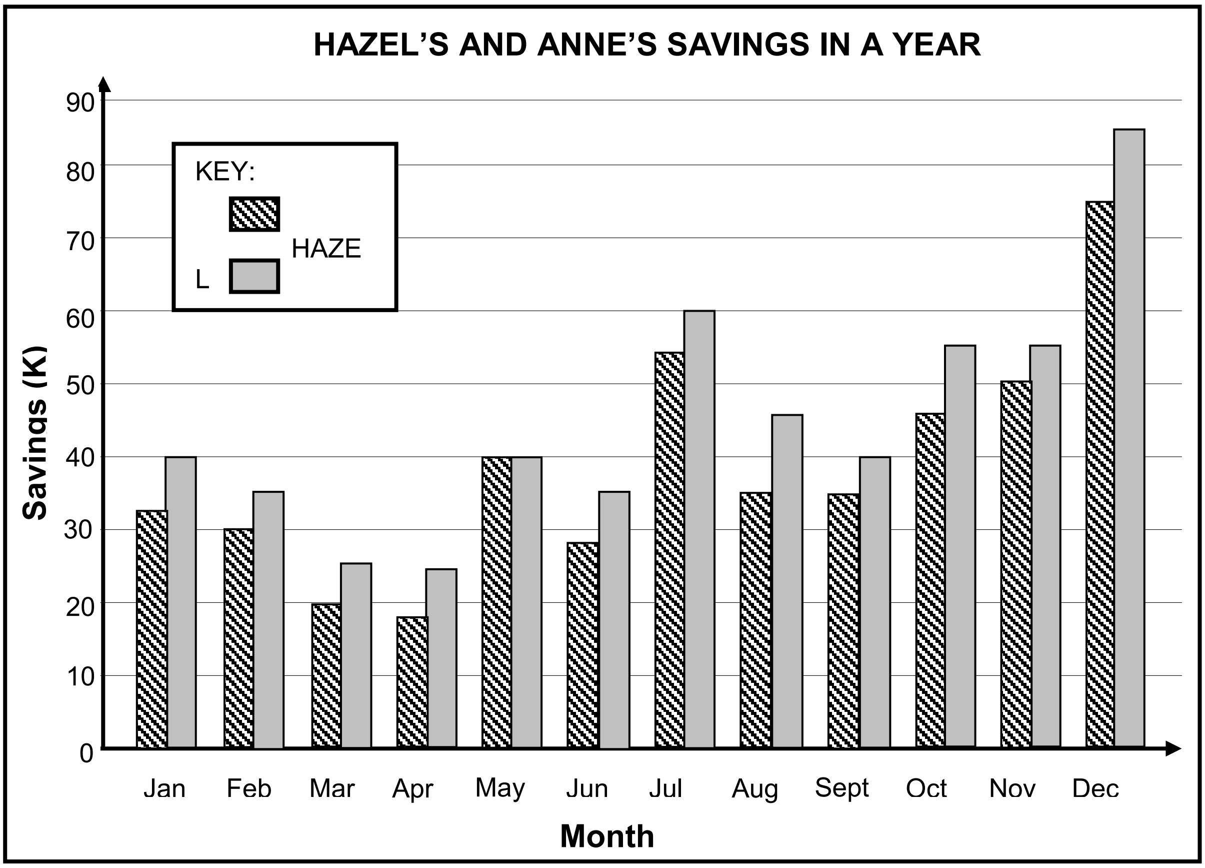
C. 1. Divided Bar Graph
2 Oil
3. 50%
4. Oil, Coal, natural gas, water power, other

Practice Exercise 3

1.



2.



Practice Exercise 4

1. A line graph is used to show **<u>changes</u>** and **<u>relationships</u>** between two sets of quantities or data. One is represented by **<u>horizontal lines</u>** on the graph and the other is represented by a **<u>vertical lines</u>**. The line segments connecting the intersections of the representations is the **<u>graph</u>** of the relationship between the two sets of quantities.

2. (a) 5900 g

   (b) 2 weeks

   (c) 5^{th} week

3. (a) 5 miles

   (b) 8 km

   (c) i. 3 km

   ii. 12 km

   iii. 9 km

   (d) i. 1 mile

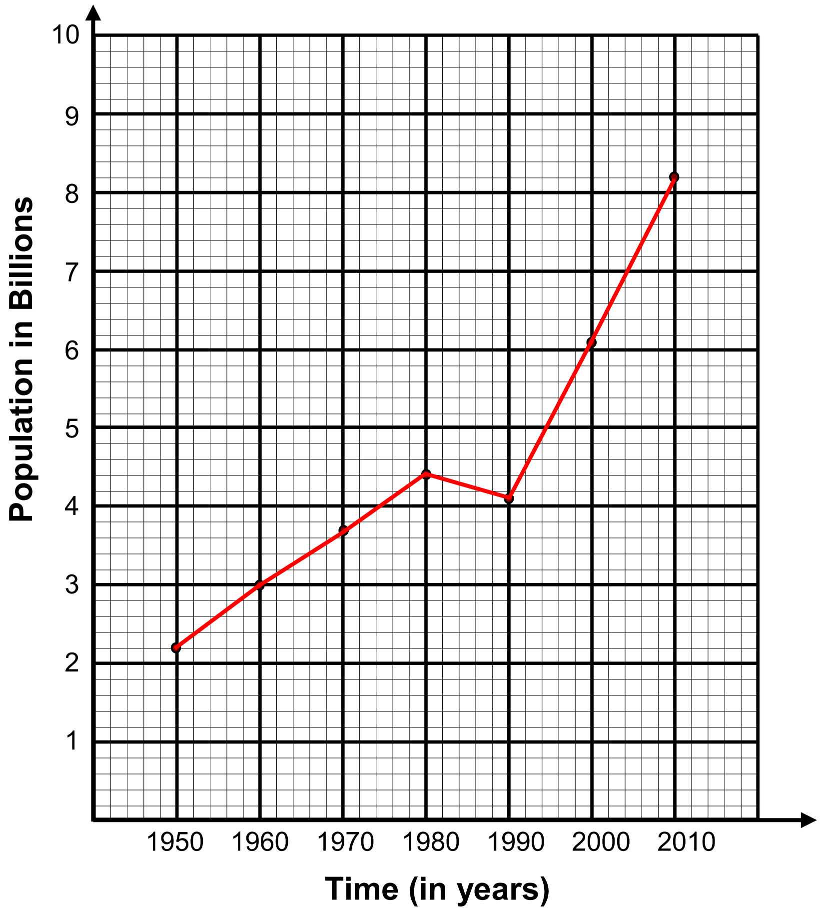
   ii. 3 miles

   iii. 7 miles

   (e) 16 km

Practice Exercise 5

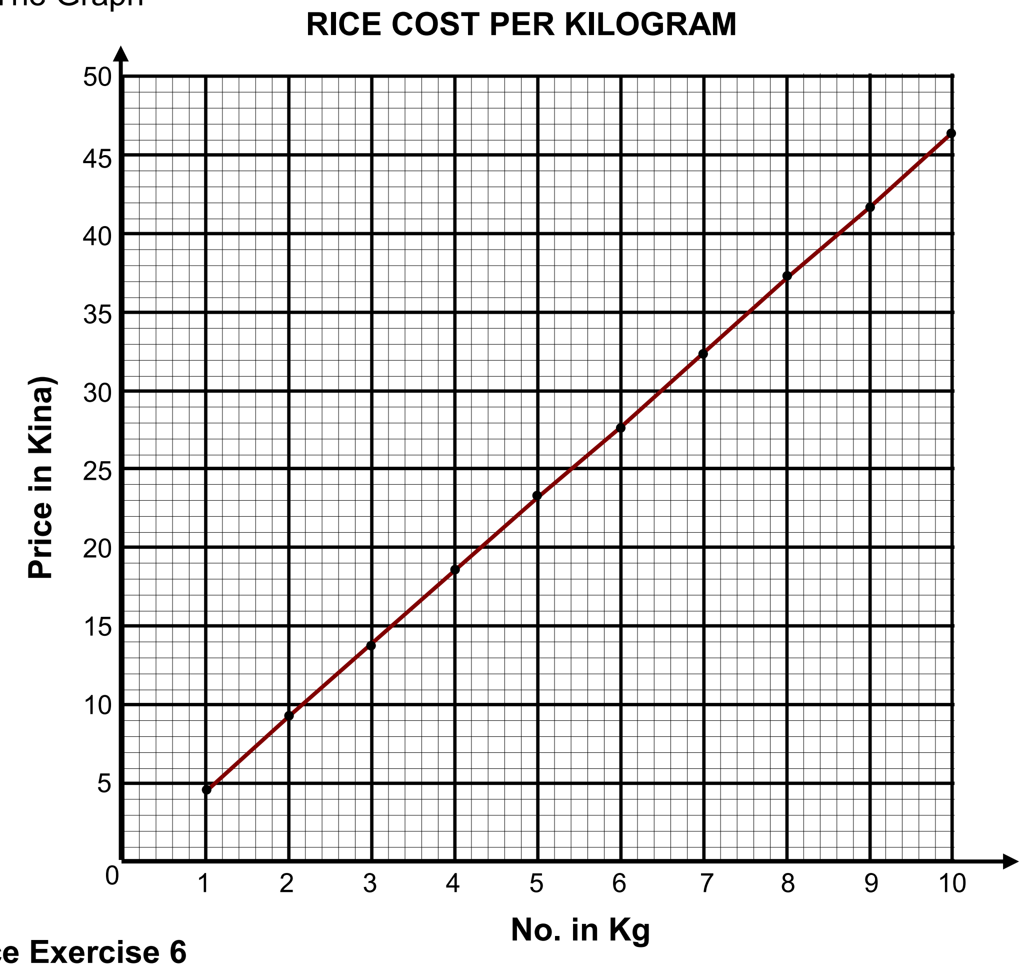
1. WORLD POPULATION GROWTH, 1950 TO 2010



2.

Number of Kilograms	Price
I	K4.65
2	K9.30
3	13.95
4	18.60
5	23.25
6	27.90
7	32.55
8	37.20
9	41.85
10	46.50

b. The Graph



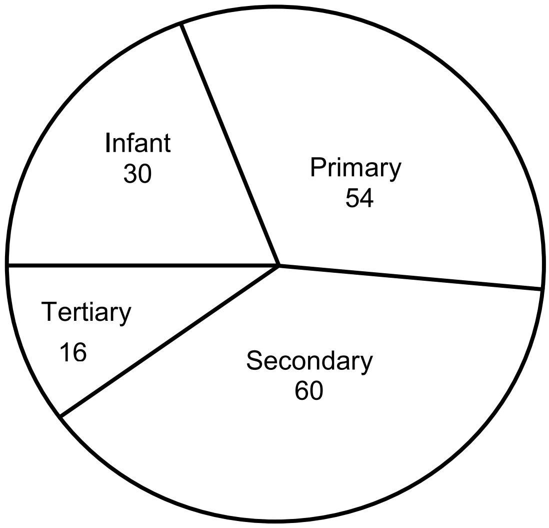
Practice Exercise 6

1. (a) Primary
   (b) 12%
   (c) 32%, 9% change of increase
   (d) between 1988 and 2000, the workforce in primary sector has change to a **decrease of 16%**, secondary sector **9% increase** and **7% increase** in the tertiary sector.

2. (a) 100%.
   (b) the boy sleep for 8 hours, play for 4 hours
   (c) A Quarter of a Day (6 hours)

Practice Exercise 7

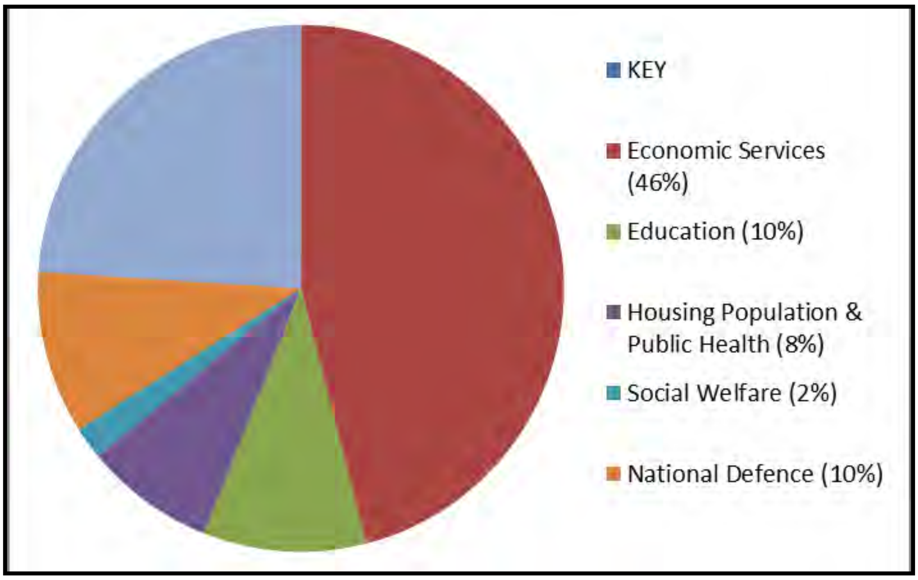
1. **MAKE-UP OF KOSETENG CENTRAL SCHOOL**



2. (a) Economic Services
   (b) Social Welfare
   (c) K6 Billion each
   (d) 3 times

The Circle Graph

NATIONAL GOVERNMENT EXPENDITURES, 1982



END OF SUB-STRAND 1

SUB STRAND 2

STATISTICS

SUB-STRAND 2: STATISTICS

Introduction

Dear student,

We welcome you to Sub-strand 2, Strand 5 of your Grade 8 Mathematics. In this sub-strand you will extend your knowledge on statistics by learning interesting and useful information.

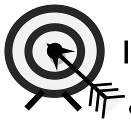
Much of this Sub-strand is a review of your Grade 7 work. You can have a look at your Grade 7 work book to make a link to the current Sub-strand.

In this Sub-strand you will study:

- the definitions of the measures of central tendencies
- how to calculate the three averages namely mean mode and median.
- how to differentiate a grouped from an ungrouped frequency data
- and read histograms accurately.

Lesson 8: Measures of Central Tendency

Researchers in the world today depend on Measures of Central Tendency to make their judgements or conclusions in their findings. It is important that we must be able to use them.



In this lesson, you will:

- revise the terms statistics, frequency, range, mean median and mode
- find the mean, median and mode of a given set of data

We talked about Measures of Central Tendency in Grade 7. The material in this lesson will be an extension of the Grade 7 work and provide more application of the measures of central tendency or measures of location.

Let us start by revising a few terms in statistics that we covered in Grade 7 and also by defining what **measures of tendency** mean.

- **Statistics** is a branch of mathematics that deals with the collection, classification, and interpretation of numerical data.
- **Frequency** is the number of times something or an event occurs.
- **Range** is the spread of Data.
- **Mode** is the score or event with the highest frequency.
- **Median** is the middle score of a set of scores that are arranged in ascending or descending numerical order.
- **Mean** is the sum of the scores divided by the total number of scores.

In our study of statistics, there are two main aspects of the data which are of interest to us. These are: The Location of the data and the Dispersion of the data. We will only consider in this section the first aspect which is the location of data. The second aspect will be dealt with in your study of higher mathematics.

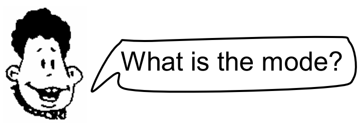
The first aspect is the location of data and the various numbers that give us information about this are called the **measures of location** or commonly known as **measures of central tendency**.

Measures of central tendency are the measures which give the location of the centre or middle of the data.

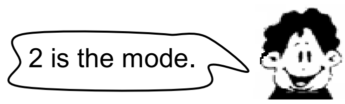
The mode, the median and the mean are three measures of central tendency. We will discuss these measures of location in the order of their increasing importance.

First let us work out the Mode in a set of Data

Here is a set of test marks scored by 10 students in a class.



2, 3, 4, 2, 6, 5, 4, 2, 6, 2



Remember the definition of MODE. It is the score that appears the most in a set of scores, 2 appeared the most in the set of scores. Therefore, 2 is the mode.

Here are some more examples

The following are scores scored by a soccer team in its matches during a season.

2, 0, 1, 1, 1, 2, 0, 0, 3, 4, 0

What is the modal score? 0 is the mode because it appeared more than the others.

Sometimes, in a set of numbers no mode exists as for the set 2, 4, 5, 0, 3, 6.

It is possible that there can be more than one mode.

Example 2, 3, 3, 3, 4, 4, 5, 6, 6, 6,7.

You can see that 3 and 6 are the modes in this set of scores because each of these numbers appears 3 times which is more than the other numbers.

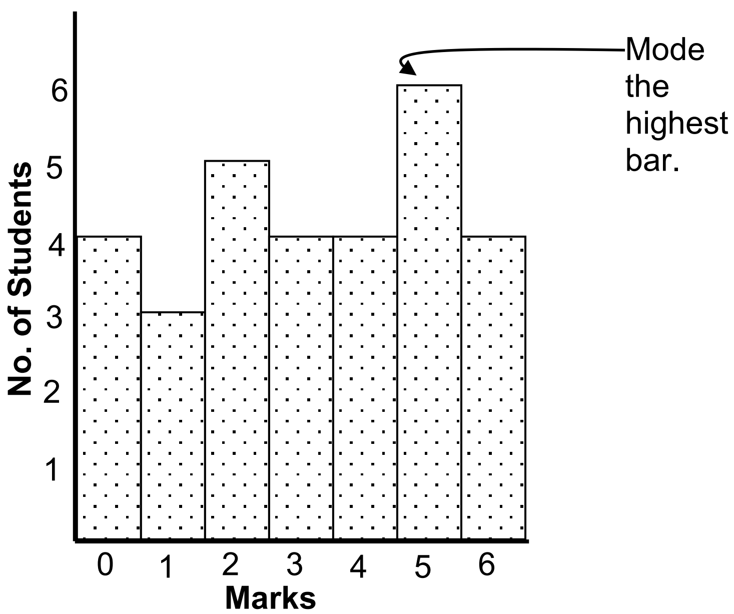
A set of values which have two modes is called **bimodal**.

A set which has one mode is called a **unimodal**.

A set which has more than two modes is called **multimodal**.

We can also work out the modes from reading graphs. Here is an example.

The Graph below shows the marks of a Mathematics Test by 30 students.



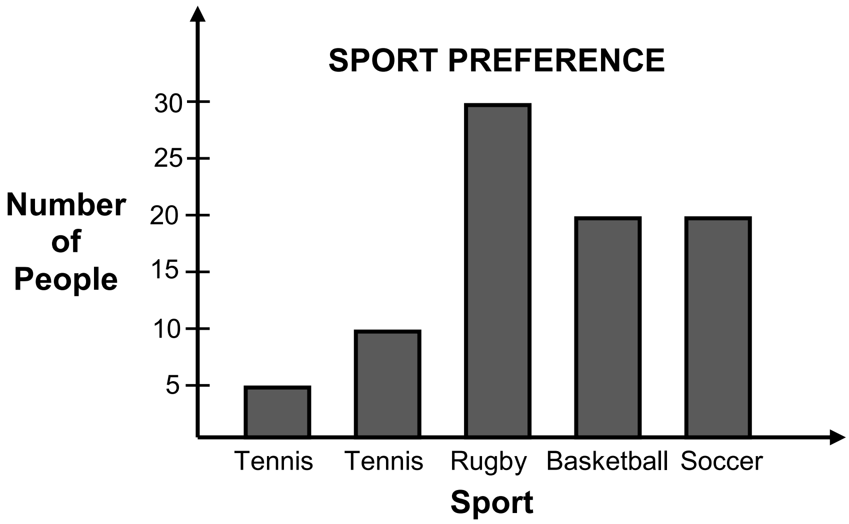
What is the most frequent mark? 5

Why 5? This is because 5 appeared often or we can say more students scored 5.

In real life Mode is used when we are asked to choose the most popular item.

Example 2

The graph shows the preferred sport for a number of people.



Modal means the same as **MODE**

What is the most preferred sport? Rugby is the most preferred sport. This means most people like rugby than the other sports.

It can also be called the modal sport.

Mean

Mean is another word for average. It is the total number of scores divided by the number of scores.

The Mean is a measure that gives the centre of a set of values. Let us have a look at some examples.

Example 1 2, 0, 1, 1, 1, 2, 0, 0, 3, 4 there are 11 with the crossed out 0

$$\text{Mean} = \frac{\text{sum of scores}}{\text{number of scores}} = \frac{14}{10} = 1.4$$

The mean score is therefore 1.4

Example 2 Test marks for 15 student in a mathematics test.

10, 11, 7, 9, 10, 6, 5, 7, 4, 6, 11, 5, 7, 13,9.

What is the mean?

Solution

$$\text{Mean} = \frac{\text{sum of scores}}{\text{number of scores}}$$

$$= \frac{10+11+7+9+10+6+5+7+4+6+11+5+7+13+9}{15}$$

$$= \frac{120}{15} = 8.$$ The mean score is 8.

Example 3

In the first 4 tests this year Solomon scored 67, 34, 54, 45. What is his mean mark?

$$\text{Mean} = \frac{\text{sum of scores}}{\text{number of scores}}$$

$$= \frac{67+34+54+45}{4}$$

$$= \frac{200}{4} = 50$$

Therefore, 50 is Solomon's mean mark.

Median

- **It is the middle score in a set of odd number of scores when the scores are arranged in order.**
- **For a set that contains an even number of scores the median is the mean of the two middle scores.**

Example 1

This is a set of marks for a class of students.

10, 11, 7, 9, 10, 6, 5, 7, 4, 6, 11, 5, 7, 13, 9

Let us work out the median score.

Step 1 Arrange the scores in order from the lowest to the highest

4, 5, 5, 6, 6, 7, 7, 7, 9, 9, 10, 10, 11, 11, 13

Step 2 Pick the middle score

4, 5, 5, 6, 6, 7, 7, 7, 9, 9, 10, 10, 11, 11, 13

Thus 7 is the median.

Example 2

The test scores for 21 students are:

15	16	18	23	22	28	29
25	24	27	18	11	20	23
26	26	30	25	18	15	17

Step 1 Arrange the scores in order from smallest to largest.

11	15	15	16	17	18	18
18	20	22	23	23	24	25
25	26	26	27	28	29	30

Step 2 Pick out the middle number

11, 15, 15, 16, 17, 18, 18, 18, 20, 22, **23**, 23, 24, 25, 25, 26, 26, 27, 28, 29, 30

The middle score is the median. **Therefore, the median is 23.**

What about the median of a set that contains an even number of scores?

Example 3

Work out the median of this set of scores.

6	8	4	3	2
9	0	1	5	3

Step 1 Arrange the scores in order from the smallest to the largest

0 1 2 3 3 4 5 6 8 9

Step 2 Pick out the middle scores

0 1 2 3 **3 4** 5 6 8 9

Step 3 Find the mean of the middle scores

$$\text{Median} = \frac{\text{The two middle scores}}{2}$$

$$= \frac{3 + 4}{2}$$

$$= 3.5$$

Therefore, the median is 3.5

Example 4

The test scores for 22 students are,

15, 16, 18, 23, 22, 28, 29, 25, 24, 27, 18
11, 20, 23, 26, 26, 30, 25, 18, 15, 17, 18

Step 1 Arrange the marks in order from the lowest to the highest.

11, 15, 15, 16, 17, 18, 18, 18, 18, 20, 22, 23, 23, 24, 25,25, 26, 26, 27, 28, 29,30.

Step2 Pick the middle scores.

11, 15, 15, 16, 17, 18, 18, 18, 18, 20, **22, 23**, 23, 24, 25,25, 26, 26, 27, 28, 29,30.

Step 3 Find the median

$$\text{Median} = \frac{22+23}{2} = 22.5$$

The median is therefore 22.5

NOW DO PRACTICE EXERCISE 8

Practice Exercise 8

1. Work out the median and the mode of the following set of scores.

   a) 1, 5, 6, 7, 1, 1, 2, 3 and 4

   b) 2, 2, 2, 0, 3, 3,3, 4, 4 and 5

   c) 8, 11, 12, 14, 15 and 18

---

2. Work out the mean of these set of scores.

   a) 2, 5, 3, 2, 6, 7, 5, 8, 10, 9

   b) 7, 8, 9, 10, 11, 12

   c) 0, 0, 1, 1, 1, 3, 3, 4, 4, 5, 6

---

3. Calculate the mean of the numbers 1, 2, 3, 5, 6 and 7.

---

4. Determine the mode of the following. 2, 5, 12, 9, 1, 7, 10, 4

---

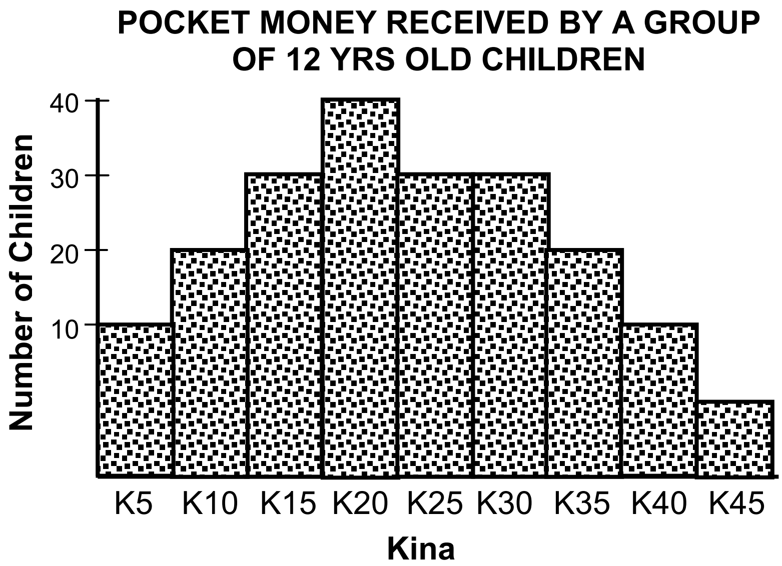
5. The wages of five office juniors are K59, K55.60, K64.80, K98.40 and K74.80 per week. Find the mean wage.

6. Work out the mean of the following amount:

K1, K2, K3.50, K5, K4, K2.50

---

7. The Histogram shows the amount of pocket money received by a group of 12 year old children.



a) What amount of money did most students get?

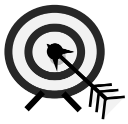
b) What is the modal amount?

c) How many students received K45?

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2.

Lesson 9: Simple or Ungrouped Frequency Distribution

You learnt about the measures of central tendency in the last lesson.



In this lesson, you will:

- define frequency, frequency table and frequency distribution
- present a set of data in a frequency table
- make a simple frequency distribution table.

Let us start this lesson by defining the following terms: frequency, frequency table and frequency distribution.

Frequency is the number of times something occurs.

The **Frequency Table** is a table showing a list of every possible score in one column and the frequency of occurrence of each score in another column.

A **Frequency Distribution** refers to a systematic technique for showing the number of observations in each of the scores or values. It is normally presented in a Frequency Table.

Let us study the tables below to fully understand the meaning of the terms explained above.

Here is an example of a **Frequency Table**. The table is a presentation of the following results of a die being tossed 20 times.

2, 1, 3, 1, 1, 4, 5, 3, 6, 3, 4, 5, 1, 2, 1, 4, 3, 5, 3, 4

1 occurred 5 times.

N is the total number of occurrence of scores

Outcomes	Frequency
1	5
2	2
3	5
4	4
5	3
6	1
N	**20**

The first column shows the possible outcomes of the throwing of the die. The next column on the right is the frequency column. The frequency of the score is simply the number of times that score appears on the set of data.

Frequency distribution tables sometimes have 3 columns.

A frequency distribution table can be prepared using the following steps:

1. Construct a table with three columns.
2. Fill in the first column. The first column shows the outcomes arranged in ascending order.
3. Fill in the second column. The second column is the tally column. Take each observation from the data, one at a time and indicate the frequency (the number of times the observation has occurred in the data) by small lines called **tally marks.** The tally marks are written in a bunch of 5, the fifth one is drawn diagonally crossing the other four marks.
4. Fill in the third column by writing the numerical value of the tally marks in the second column.
5. Add the frequencies and give a total of the frequencies at the end or bottom of the column.

Let us follow the steps to draw a frequency distribution table.

The following are quiz results of 20 students. Use the results to complete a frequency distribution table.

7	8	5	4	9	8	5	7	6	8
9	6	7	9	8	7	9	9	4	6

Step 1: Construct the table with 3 columns.

Marks	Tally	Frequency

Step 2: Fill in the first column. The first column shows the outcomes arranged in ascending order.

Marks	Tally	Frequency
4		
5		
6		
7		
8		
9		

Step 3: Fill in the second column. The second column is the tally column. Take each observation from the data, one at a time and indicate the frequency (the number of times the observation has occurred in the data) by small lines called tally marks. The tally marks are written in a bunch of 5, the fifth one crosses the four other marks diagonally.

Marks	Tally	Frequency
4	$\vert\vert$	
5	$\vert\vert$	
6	$\vert\vert\vert$	
7	$\vert\vert\vert\vert$	
8	$\vert\vert\vert\vert$	
9	$\cancel{\vert\vert\vert\vert}$	

Step 4: Fill in the third column by counting the tally mark in the second column and writing the numerical value.

Marks	Tally	Frequency
4	$\vert\vert$	2
5	$\vert\vert$	2
6	$\vert\vert\vert$	3
7	$\vert\vert\vert\vert$	4
8	$\vert\vert\vert\vert$	4
9	$\cancel{\vert\vert\vert\vert}$	5

Step 5: Add the frequencies and give a total of the frequencies at the end or bottom of the column.

Marks	Tally	Frequency
4	$\vert\vert$	2
5	$\vert\vert$	2
6	$\vert\vert\vert$	3
7	$\vert\vert\vert\vert$	4
8	$\vert\vert\vert\vert$	4
9	$\cancel{\vert\vert\vert\vert}$	5
	Total	20

Using the 5 steps let us draw a frequency distribution table by filling in the following test scores

6	10	11	6	10	10	10	11	7	7
8	8	9	11	10	6	11	10	9	9

Step 1

Scores	Tally	Frequency

Step 2

Scores	Tally	Frequency
6		
7		
8		
9		
10		
11		

Step 3

Scores	Tally	Frequency
6	III	
7	II	
8	II	
9	III	
10	~~IIII~~ I	
11	IIII	

Step 4

Scores	Tally	Frequency
6	III	3
7	II	2
8	II	2
9	III	3
10	~~IIII~~ I	6
11	IIII	4

Step 5

Scores	Tally	Frequency
6	III	3
7	II	2
8	II	2
9	III	3
10	~~IIII~~ I	6
11	IIII	4
	Total	20

NOW DO PRACTICE EXERCISE 9

Practice Exercise 9

1. Define the following:

   a) Frequency

   b) Frequency Table

   c) Frequency Distribution

2. Given the scores 2, 3, 2, 4, 3, 2, 1, 3, what is the frequency of 3?

3. What is the mode of the set of scores in Question 2?

4. Work out the median.

5. Calculate the mean.

Refer to the following scores to answer items a to d.

0	1	3	2	1	1	3
4	2	1	3	2	1	0

a) Complete the Frequency Distribution table.

Scores	Tally	Frequency

b) From the Frequency distribution table, what is the modal score?

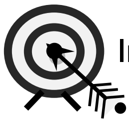
c) Work out the median.

d) Calculate the mean score.

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2.

Lesson 10: Finding the Mean from a Frequency Distribution Table

You learnt how to make a frequency distribution table in the last lesson.



In this lesson, you will:

- prepare a frequency distribution table by following the steps accurately
- find the mean from a frequency distribution table.

Let us revise the steps used in preparing a frequency distribution table.

Step 1: Construct a table with three columns.
Step 2: Fill in the first column by writing the scores in ascending order.
Step 3: Fill in the second column with tally marks.
Step 4: Fill in the third column with respective frequencies.
Step 5: Total the frequencies.

Let us now follow the steps to construct a frequency distribution table.

Example 1

The following scores are student marks in a test out of 10.

9	5	6	4	3	2	3	4	5
7	8	3	2	5	6	7	5	6

Score(x)	Tally	Frequency
2	II	2
3	III	3
4	II	2
5	IIII	4
6	III	3
7	II	2
8	I	1
9	I	1

Frequency distribution tables can be used to work out the mean of a set of data. To work out the mean from the Frequency Distribution table, we will require another column called the fx column. It is pronounced as "**ef ex**".

The fx column gives the product of column 1 and 3 that is the product of the score and its corresponding frequency.

See how the fx column is completed in Example 1.

Score(x)	Frequency(f)	fx
2	2	2 x 2 = 4
3	3	3 x 3 = 9
4	2	4 x 2 = 8
5	4	5 x 4 = 20
6	3	6 x 3 = 18
7	2	7 x 2 = 14
8	1	8 x 1 = 8
9	1	9 x 1 = 9
	N = 18	$\sum$**fx** = 90

Score (x) times frequency (f) 2 x 2 = 4

Sum of fx column

Once the extra column is drawn and filled as shown above we can then work out the mean of the data following the rule

$$\textbf{Mean} = \frac{\text{Sum of the products of each score x corresponding frequency}}{\text{Sum of all frequencies}}$$

This can also be expressed in symbols

$$\bar{\mathbf{X}} = \frac{\sum \mathbf{fx}}{\mathbf{N}}$$

Where: $\bar{\mathbf{X}}$ stands for the mean

Σfx is the sum of the fx columN

N is the sum of all the frequencies

Let us now use the rule to work out the mean of the set of data in the table above.

$$\bar{\mathbf{X}} = \frac{\sum \text{fx}}{\text{N}}$$

$$= \frac{90}{18} = \mathbf{5}$$

Therefore, the mean is equal to 5.

Example 2

The following table show the results of a die being rolled 60 times.

Score(x)	Frequency(f)	fx
1	12	12
2	11	22
3	8	24
4	12	48
5	7	35
6	10	60
Total	60	201

Using the table let us find the mean of the set of data

$$\overline{\mathbf{X}} = \frac{\sum \mathbf{fx}}{\mathbf{N}}$$

$$\overline{\mathbf{X}} = \frac{201}{60} = 3.35$$

The mean of the experiment of rolling a die 60 times is 3.35

Example 3

A hockey team plays 15 matches. Below is a list of goals scored in each match

1, 0, 2, 4, 0, 1, 1, 1, 2, 5, 3, 0, 1, 2, 2

The scores are displayed on a frequency distribution table below

Score(x)	Frequency(f)	fx
0	3	0
1	5	5
2	4	8
3	1	3
4	1	4
5	1	5
Total	15	25

What is the mean number of goals?

$$\overline{\mathbf{X}} = \frac{\sum \mathbf{fx}}{\mathbf{N}}$$

$$= \frac{25}{15}$$

$$= 1.67$$

Example 4

The ages of a group of girls are 14, 14, 13, 14, 14, 14, 14, 13, 13, 12, 12,13.

The ages are filled in a frequency distribution table shown below.

Ages(x)	**Frequency(f)**	**fx**
12	2	24
13	4	52
14	6	84
Total	12	160

Now let us work out the mean age of the girls.

$$\overline{\mathbf{X}} = \frac{\sum \mathbf{fx}}{\mathbf{N}}$$

$$= \frac{160}{12}$$

$$= 13.3$$

The mean age of the girls is 13.3.

From the same frequency distribution, what is the modal age?

Answer: 14 years old is the modal age, six girls are 14 years old or most girls in the group are 14 years old.

NOW DO PRACTICE EXERCISE 10

Practice Exercise 10

1. The total scores when two dice are thrown 20 times are: 7, 4, 5, 7, 3, 2, 8, 6, 8, 7, 6, 5, 9, 7, 3, 8, 7, 6, 5, 2.

   a) Construct a Frequency distribution table of the scores

Scores(x)	Tally	Frequency (f)

   b) Find the mean of the set of scores

2. The number of girls present in a grade over a two week period is:

   12, 14, 12, 14, 13, 11, 14, 11, 10, 12

   a) Construct a frequency distribution table.

Scores(x)	Tally	Frequency (f)

   b) Find the mean of the days present.

3. Sixty hibiscus flowers are planted. At their flowering peak the flowers per hibiscus tree were counted.

   The results are shown below.

Flowers	0	1	2	3	4	5	6	7	8
Frequency	0	0	0	6	4	6	10	16	18

   a) Find the mean number of flowers per hibiscus tree.

   b) What is the modal number of flowers? ________________________

4. An industrial organisation gives an aptitude test to all applicants for employment. The results for the 150 people taking the test were:

Score	Frequency
1	6
2	12
3	15
4	21
5	35
6	24
7	20
8	10
9	6
10	1

a) Calculate the mean score.

b) What is the modal score?

c) How many students scored the highest mark?

d) How many students scored the lowest mark?

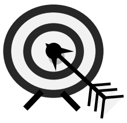
---

5. The mean mass of 5 basketball players is 120.5 kg. The mean mass plus a substitute is 150.5 kg. Calculate the mass of the substitute.

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB – STRAND 2.

Lesson 11: Grouped Frequency Distribution

In the previous lessons we dealt with filling in frequency distribution tables of ungrouped data



In this lesson, you will:

- Extend the above ideas onto grouped data describing grouped frequency distribution
- recognise why it becomes necessary to group data
- describe the range, class size, class intervals and midpoint or class centre for grouped data
- prepare a grouped frequency distribution table

What is a grouped frequency distribution?

A grouped frequency distribution is obtained when the scores are classed into groups or intervals because they are too many to represent individually. All the scores in the particular class are then counted as making up the frequency in that class.

Unlike a Frequency distribution of ungrouped data, a grouped distribution deals with a much larger amount of numerical data. Because the amount of data is too large it becomes necessary to group the numbers into classes or categories.

We can then find the number of items belonging to each class thus obtaining a class frequency.

The main advantage of grouping is that it produces a clear overall picture of the distribution. However, too many groups will destroy the pattern of the distribution whilst too few will destroy much of the detail which was present in the raw data.

To draw a frequency distribution we need to calculate the range, class size, class intervals, and midpoint or class centre. Let us use the table below to explain each of the above terms.

The numbers shown are times in seconds for 40 children to complete a length of a swimming pool. The swimmers were divided into heats as the pool had 8 lanes.

	Lane number							
Table	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
I	40	49	43	35	42	43	46	36
II	42	36	37	44	39	42	32	46
III	39	49	45	52	39	54	36	33
IV	30	44	32	53	47	44	51	42
V	40	42	49	48	33	53	48	43

First we calculate the range.

The range is the spread of time from the smallest to the largest. To work out the range subtract the lowest from the highest score.

Example: **Range = highest time – lowest time**

= 54 – 30

= 24

Next we calculate the class.

Class is another word for group, depending on volume of data, the number of classes is usually less than 20

The class size is the number of the members of the group which is determined by dividing the range by the number of preferred class.

Example

If we choose to have 5 classes then the class size is:

$$\text{Class size} = \frac{\text{Range}}{\text{Number of classes}}$$

$$= \frac{24}{5} = 4.8$$

The **class centre or the mid- point** have the same meaning. The class centre refers to the midpoint of the class.

Example:

Using the class interval 30 – 34, this interval represents the times from 30 to 34 that is 30, 31, **32,** 33, 34

The midpoint or the class centre is 32. It is the middle score.

Sometimes the class centre is worked out by adding the end class limits and dividing by 2.

$$\textbf{Class centre} = \frac{\textbf{upper class limit + lower class limit}}{\textbf{2}}$$

Example: Referring to the class interval 30 – 34 . Its class centre is,

$$\text{Class centre} = \frac{\text{upper class limit + lower class limit}}{2}$$

$$= \frac{30 + 34}{2} = 32$$

The class centre is 32.

Let us now, draw a grouped frequency distribution representing the information on the swimmers time.

Remember, the class size is 5. So, the classes start from 30 – 34 and so on.

Class	Frequency
30 - 34	5
35 - 39	9
40 - 44	12
45 - 49	8
50 - 54	6
Total	40

Example 2

The following are ages of workers in a certain factory

16	16	16	16	17	17	17	17	18	18
18	18	19	19	19	19	20	20	20	20
21	21	22	22	23	23	24	24	25	25
25	25	25	25	25	25	25	25	26	26
26	26	27	27	27	27	27	28	28	29
29	30	31	31	31	31	31	32	32	32
32	33	34	35	36	36	37	38	39	40

Let us now organise the data on a frequency distribution table.

Step 1 Identify the range, the ages of the workers ranges from 16 to 40
Range = highest age – lowest age

= 40 – 16 = 24

Step 2 Identify the class size, if the number of classes is 5

$$\text{Class size is} = \frac{\text{range}}{\text{number of classes}}$$

$$= \frac{24}{5} = 4.8$$

Each class will have either 4 or 5 members. If we choose to have 5 members then the classes will have the following class intervals

16 – 20 21- 25 26- 30 31 – 35 36 – 40

We can now construct the frequency distribution table

Age group	Frequency
16 – 20	20
21- 25	18
26- 30	14
31 – 35	12
36 – 40	6
Total	70

Information can be acquired once a frequency distribution table is formed.

Example

1. What is the total number of workers?

   Answer: 70 workers, we can obtain the answer from the total frequency column

2. What is the oldest age group? **Answer:** 36 – 40

3. How many people are in the youngest age group? **Answer**: 20 people

Note: The shaded parts of the table give the answers to the above three questions.

NOW DO PRACTICE EXERCISE 11

Practice Exercise 11

1. Define class centre.

2. Is midpoint the same as class centre? _____.

3. State the reason for using grouped frequency distribution as opposed to ungrouped frequency distribution.

4. The following are hours of overtime for a group of workers.

1 2 3 4 6 7 8 9 2 3 5 10 11 12 13 14
16 17 18 19 10 12 13 14 15 16 17 10 18 19
20 21 23 24 25 26 27 28 30 29 20 21 22…23
24 23 24 27 24 23 25 26 25

a) Calculate the range

b) Workout the class size if the number of classes is 5

c) Construct a grouped frequency distribution.

Hours of Over time	Frequency

d) From the frequency distribution, what is the modal class?

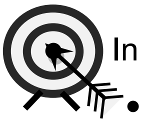
e) How many people worked for less than 7 hours?

f) How many people worked for more than 24 hours?

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2

Lesson 12: Histogram and Frequency Polygons

In the previous lesson we described grouped frequency distributions and constructed frequency distribution tables.



In this lesson you will,

- describe histograms and frequency polygons
- construct a histogram and a frequency polygon for a set of data
- interpret information presented statistically.

This lesson is a revision to what you studied in Grade 7.

Let us revise the description of a histogram and a frequency polygon.

The histogram is a special type of bar graph where the bars are always vertical and are placed next to each other without gaps. The class marks are plotted on the horizontal axis and the frequency on the vertical axis.

A histogram for a grouped distribution can be drawn by using the midpoints of the class intervals as centres of the bars.

Here is frequency distribution table of a grouped frequency table that can be used to draw a Histogram.

Age group	Frequency
16 – 20	20
21- 25	18
26- 30	14
31 – 35	12
36 – 40	6
Total	70

To draw a histogram we need to work out the mid points or class centres of the age group.

Recalling the last lesson, to work out the class centre add the end points and divide by 2.

Example

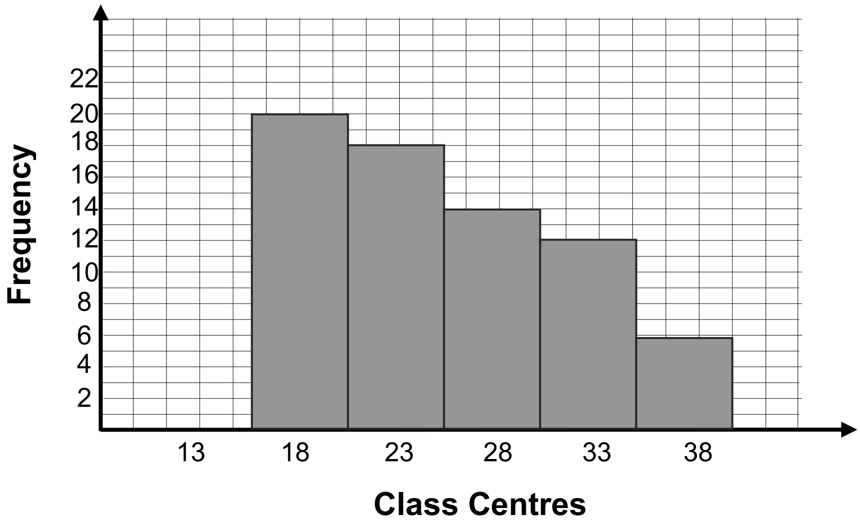
The class centre for the age group 16 – 20 is $\frac{16 + 20}{2} = 18$

The class centre for the age group 21 – 25 is $\frac{21 + 25}{2} = 23$

Here is the same table on page 83 showing the class centres of each class.

Age group	Class centre	Frequency
16 – 20	18	20
21- 25	23	18
26- 30	28	14
31 – 35	33	12
36 – 40	38	6
	Total	70

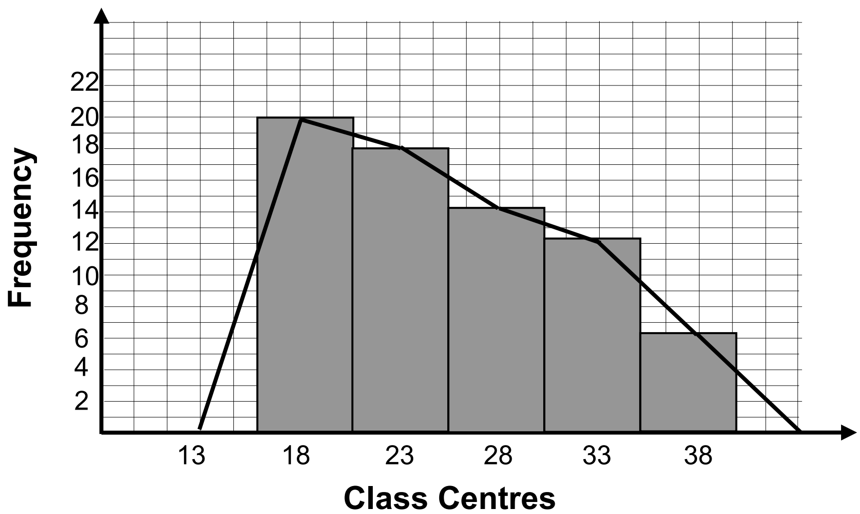
Once the class centres are known a histogram can be drawn. Below is a histogram drawn from the distribution table above.



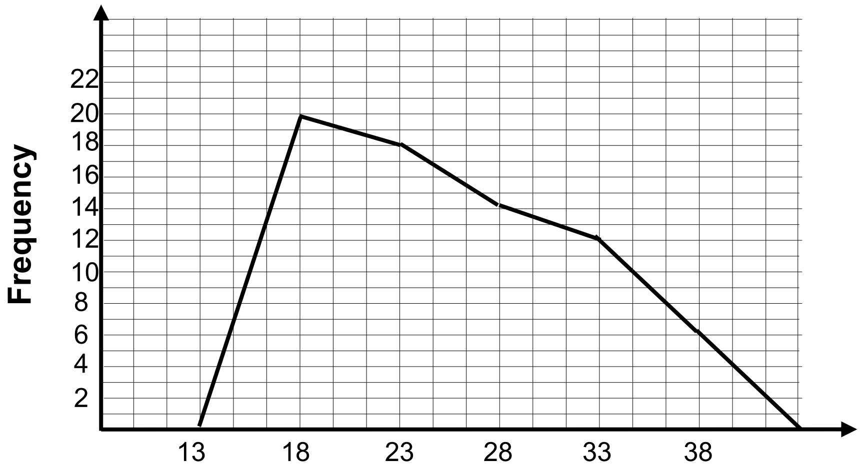
What is a Frequency Polygon?

A frequency polygon is another way of representing a frequency distribution. It is drawn by connecting the mid points of the tops of the bars in a histogram by straight lines.

Example



Without the frequency histogram a frequency polygon looks like this.



Example 2

The following table shows the swimming times for some students. Use the information to draw a histogram and a polygon.

Class	Frequency
30 - 34	5
35 - 39	9
40 - 44	12
45 - 49	8
50 - 54	6
Total	40

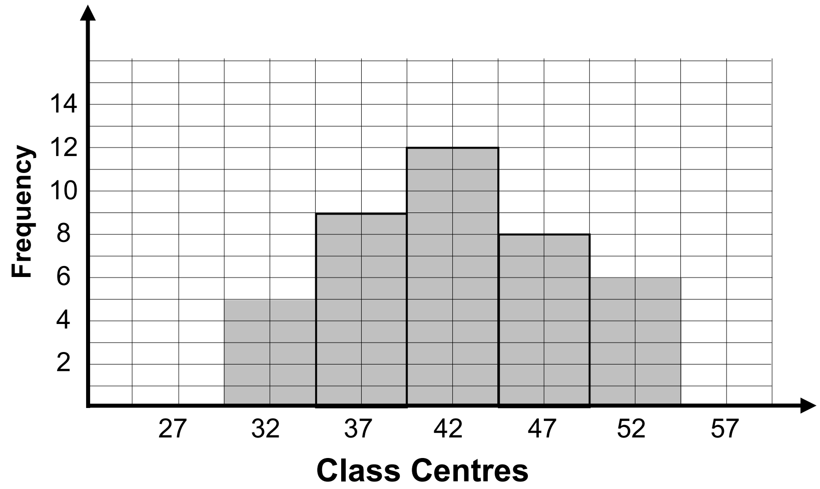
To draw a histogram and a polygon we need to find the midpoint of each class.

Class	**Class centre**	**Frequency**
30 - 34	32	5
35 - 39	37	9
40 - 44	42	12
45 - 49	47	8
50 - 54	52	6
	Total	40

A frequency histogram can be drawn once the midpoints/class centres are calculated.

Turn to the next page and see how a histogram and a frequency polygon are drawn from the frequency distribution.

HISTOGRAM OF STUDENT'S SWIMMING TIME



From the histogram the midpoints of the bars are joined to form a frequency polygon.

FREQUENCY POLYGON OF STUDENT'S SWIMMING TIME

NOW DO PRACTICE EXERCISE 12

Practice Exercise 12

Following is a record of the percentage marks obtained by 100 students in an examination:

45	93	35	56	16	50	63	30	86	65
57	39	44	75	25	45	74	93	84	25
77	28	54	50	85	55	34	50	57	55
48	78	15	27	79	68	26	66	80	91
62	67	52	50	75	36	83	20	45	71
63	51	40	46	61	62	67	57	53	45
51	40	46	31	54	67	66	52	49	54
55	52	56	59	38	52	43	55	51	47
54	56	42	53	40	51	58	52	27	56
42	86	50	31	61	33	36	56	17	25

a) Prepare a grouped Frequency distribution Table for the above scores for the classes 15 – 22, 23 – 30 and so on

b) Work out the midpoint for each interval. The first few have been done for you.

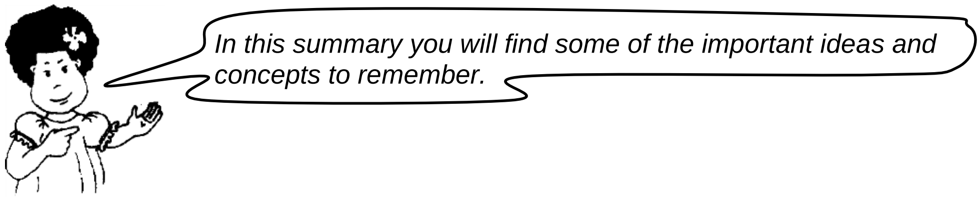
Percentage marks	Midpoint	Frequency
15 – 22	18.5	
23 – 30	26.5	
31 – 38	34.5	
39 – 46	42.5	

c) Construct a Histogram

d) Construct a Frequency polygon

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 2

SUB-STRAND 2: SUMMARY



- Statistics is a branch in Mathematics that deals with the break down and explanation of numerical data
- Range measures the spread of data. It is worked out by finding the difference between the lowest and the highest score.
- Mode is the score with the highest frequency
- Median is the middle score of a set of odd number of data arranged in descending or ascending order
- Median of a set of even number of data is calculated by adding the two middle scores and dividing it by 2.
- Mean is the average of a set of scores. It is worked out dividing the sum of scores by the number of scores
- There are five steps involved in drawing a frequency distribution table. They are:
  Step 1: Construct a table with three columns
  Step 2: Fill in the scores (x) in the first column.
  Step 3: Fill in tally marks in the second column.
  Step 4: Tally the frequency and write its numerical value in the third column
  Step 5: Total the frequencies.
- Mean can be calculated from a Frequency Distribution table using the rule

$$\overline{X} = \frac{\sum fx}{\sum f}$$

- A large number of data become difficult to present on a frequency distribution table. The data should therefore be grouped into classes.
- Classes in a grouped frequency distribution can be no more than 20
- To work out the class size, the range must be calculated first, the range is then divided by the preferred number of classes to determine the class size.
- A histogram represents information from a frequency distribution. It is a vertical bar graph. The bars are drawn with no space in between.
- A frequency polygon represents information from the frequency distribution. It is a graph that is drawn by straight line segments connecting the midpoints of the tops of the vertical bars of a histogram.

REVISE LESSONS 8 - 12 THEN DO SUB-STRAND TEST 2 IN ASSIGNMENT 5

ANSWERS TO PRACTICE EXERCISES 8 - 12

Practice Exercise 8

1. a) mode = 1 median = 3
   b) mode = 2 median = 3
   c) mode = none median =13

2. a) 5.7 b) 9.5 c) 2.54

3. 4

4. none

5. K70.52

6. K3

7. a) K20. b) K20 c) 5 students

Practice Exercise 9

1. (a) Frequency is the number of times something occurs.
   (b) Frequency Table is a table showing a list of every possible score in one column and the frequency of occurrence of each score in another column.
   (c) Frequency Distribution table, it is an organised data arranged in a category.

2. Frequency of 3 is 3
3. Mode is 2 and 3
4. Median is 2.5
5. Mean is 2.5
6.

Scores	Tally	Frequency
0	I	2
1	~~IIII~~	5
2	III	3
3	III	3
4	I	1
Total		14

(a) Modal score is 1 (b) Median is 1 (c) Mean is 1

Practice Exercise 10

1 a)

Scores (x)	Frequency(f)	fx
2	2	4
3	2	6
4	1	4
5	3	15
6	3	18
7	5	35
8	3	24
9	1	9
Total	20	115

b) Mean = $\frac{115}{20}$ = 5.75

2 a)

Days present(x)	Frequency(f)	fx
10	1	10
11	2	22
12	3	36
13	1	13
14	3	42
Total	10	123

b) Mean = $\frac{123}{10}$ = 12.3

3.

Flowers (x)	Frequency(f)	fx
0	0	0
1	0	0
2	0	0
3	6	18
4	4	16
5	6	30
6	10	60
7	16	112
8	18	144
Total	60	380

a) Mean = $\frac{380}{60}$ = 6.3 b) Modal number of flowers is 8

4.

Score(x)	Frequency(f)	fx
1	6	6
2	12	24
3	15	45
4	21	84
5	35	175
6	24	144
7	20	140
8	10	80
9	6	54
10	1	10
Total	150	762

a) Mean Score = $\frac{762}{150}$ = 5.08 b) Modal score = 5

c) 1 d) 6

5. 30 kg

Practice Exercise 11

1. The class centre refers to the midpoint of the class

2. The **class centre or the mid-point** means the same. The class centre refers to the midpoint of the class.

3. The main advantage of grouping is that it produces a clear overall picture of the distribution. It is useful when a data is too large to present.

4. a) Range is 24. b) Class size = 4.8 therefore the class size is 5.
c)

Hours of Over time	Frequency
1 - 6	8
7 – 12	9
13- 18	11
19 - 24	15
25 - 30	10
Total	53

d) 19 – 24 is the modal class
e) 8 people
f) 10 people

Practice Exercise 12

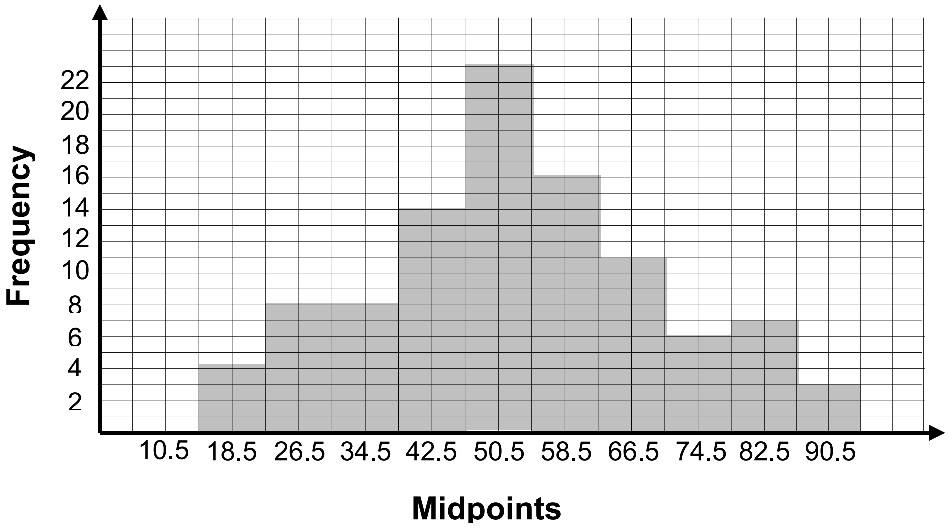
a)

Percentage marks	Frequency
15-22	4
23 - 30	8
31 - 38	8
39 - 46	14
47 - 54	23
55 - 62	16
63 - 70	11
71 – 78	6
79 – 86	7
87 - 94	3
Total	**100**

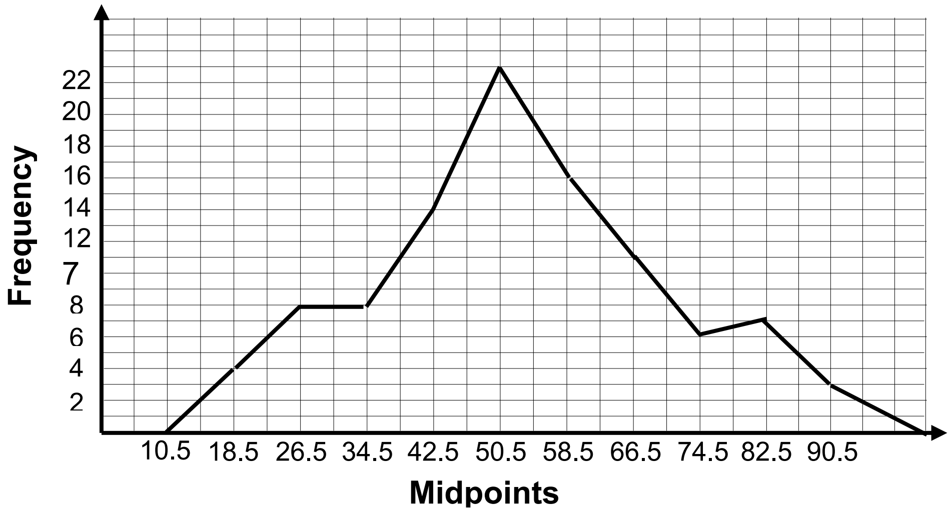
b)

Percentage marks	Midpoint	Frequency
15-22	18.5	4
23 - 30	26.5	8
31 - 38	34.5	8
39 - 46	42.5	14
47 - 54	50.5	23
55 - 62	58.5	16
63 - 70	66.5	11
71 – 78	74.5	6
79 – 86	82.5	7
87 - 94	90.5	3

c)



d)



END OF SUB-STRAND 2

SUB STRAND 3

SETS

SUB-STRAND 3: SETS

Introduction

Dear student,

Welcome to sub-strand 3 of your Grade 8 Mathematics Strand 5. In this sub-strand you will extend your knowledge of the concepts and ideas of sets

In our daily activities, we deal with sets. Sets are all around us. You do not find sets only in the classroom but it is a part of you wherever you are.

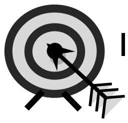
This sub strand will help you to understand what sets are and how sets are used in real life situations.

In this sub strand, you will

- explore different types of sets
- identify empty sets from givens sets
- identify union and intersection sets
- use Venn diagrams to help you solve problems. Venn diagrams will help you to understand clearly the problem that you are trying to solve.

Lesson 13: Sets

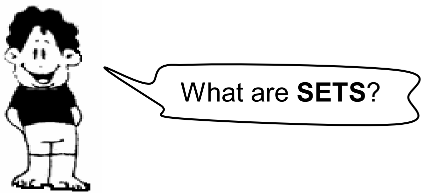
You have studied sets in Grade 7. You will extend your knowledge further s on the concepts of sets that you have learnt so far.



In this lesson, you will:

- define a set
- identify elements of a set
- identify different kinds of sets
- differentiate an empty set from other kinds of sets.

There are sets all around us. You see sets of plates or sets of cups in your kitchen, sets of uniform in the cupboard, sets of books in the classroom and many more.



A set is a collection of things or objects. The objects which belong to a set are called its elements or members.
Members or elements of a set are usually written inside braces { }.

The braces { } is used to list or describe elements in and are often denoted by a capital letter.

See the following examples

a) **A** = {2, 4, 6, 8, 10}

This set is given the name **A** and is a set of the multiples of 2 from 1 to 10.

The multiples of 2 written in the brace are called **elements** or **members** of the set.

b) We can say students in class 8A form a set because they all belong to 8A. They are members of the same class.

c) A set of drinking glasses in the cupboard form a set.

In any set that is described there are members or elements.

Example

a) Students in your class whose names begin with A

S = {Andrew, Allan, Angela, Augustine, Anton}

b) Children in your family

C = {Mary, Peter, Betty, Pawa}

Mary, Peter, Betty and Pawa are members of the set.

There are many different types of set. The following are the types of sets we will study in this lesson.

1) Universal set
2) Subset
3) Finite sets
4) Infinite sets
5) Equal sets
6) Equivalent sets
7) Empty set or the null set

- **A universal set consists of all members or elements considered in a given problem.**
- **A universal set is denoted by U or X in some books.**
- **A subset consists of either some or all of the members that belong to a universal set.**

Example 1

Set A consists of counting numbers from 1 to 10. It is called a universal set.

A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Set B consists of counting odd numbers from 1 to 10. It is a subset of the universal set.

B = {1, 3, 5, 7, 9}

All the members in set B are members of set A, which is the universal set.

Example 2

Suppose set C is the set of Mr. Wai"s children. It is a universal set.

Set C = {John, Mary, Peter, Moses, Aaron, Betty}

If set D = {Mary, Betty}, then set D is a subset of set C, which is the universal set.

The Empty set sometimes called the Null set is a set that contains no members. It is denoted by Ø.

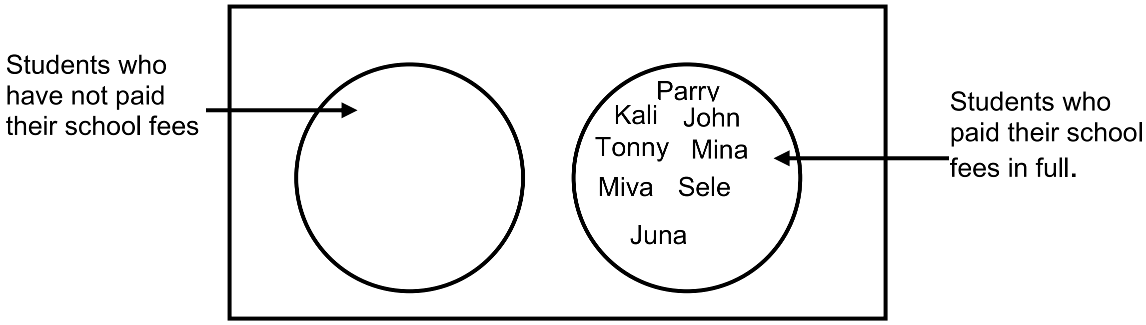
Example 1

Set E = { } or Set F = Ø

We can say an empty set has no members or elements. An empty set is one that has a description but there are no elements belonging to it.

This diagram can help you understand clearly.

The Set represents students in 8A.



Students in class 8A are in the set "students who paid their school fees in full". The set "students who have not paid their school fees in full do not have any members, so it is an Empty set.

Here are few more examples.

1. List a set of the Months of the year.

   M = {Jan, Feb, Mar, Apr, May, Jun, Jul, Aug, Sept, Oct, Nov, Dec}

2. From set M. List a set of months that begin with J.

   J = {June, July}

3. List a set of months that begin with B.

   B = { } Set B is an Empty set. There are no elements in it because we do not have months that start with the letter B.

   So Set B is a null set

- **A finite set is a set whose members or elements can be listed.**
- **An Infinite set is a set whose members or elements cannot be listed.**

Example 1

Set A is the set of counting numbers from 1 to 10.

A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

All members of the set of counting numbers from 1 to 10 are easily listed therefore the set is called a finite set.

Set B is a set which contains counting numbers.

B = {1, 2, 3, 4, 5, …}

Set B contains all the counting numbers. It is difficult to list all the counting numbers. If we were to list all the numbers we would run out of space and eventually would not be able to list all of them. Such a set is therefore, referred to as an infinite set.

- **Equal sets have the same members or elements.**
- **Equivalent sets have same number of elements**

Example 1

A = {1, 2, 3, 4, 5, 6} B = {1, 2, 3, 4, 5, 6}

Set A and B are equal sets, because their elements are the same.

C = {a, b, c, d, e, f} D = {1, 2, 3, 4, 5, 6}

Set C and D are equivalent sets because the number of elements in each set is the same, that is 6.

NOW DO PRACTICE EXERCISE 13

Practice Exercise 13

1. List the elements of the following sets.

   a) Students in your class from Central Province

   ____

   b) Names of children in your family

   ____

   c) Provinces in the country

   ____

   d) Months in a year

   ____

   e) Subjects taken in Grade 8

   ____

2. Describe each of the sets.

   a) red emperor, mackerel, tuna, barramundi

   ____

   b) sweet potato, banana, kaukau, taro

   ____

   c) sugar cane, pineapple, mandarin, pawpaw

   ____

   d) Tavurvur, Lamington, Vulcan, Mt. Ulawun

   ____

   e) a, e, i, o, u

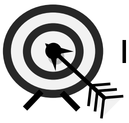
   ____

3. Draw an example of an empty set from the months in a year.

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3

Lesson 14: The Venn Diagram

You learnt the meaning of a set and saw examples of the different types of sets in the previous lesson.



In this lesson, you will:

- describe and draw Venn diagrams
- represent sets on a Venn diagram.

You will need to revise ideas in sets, subsets, empty and universal sets from the last lesson in order to understand this lesson.

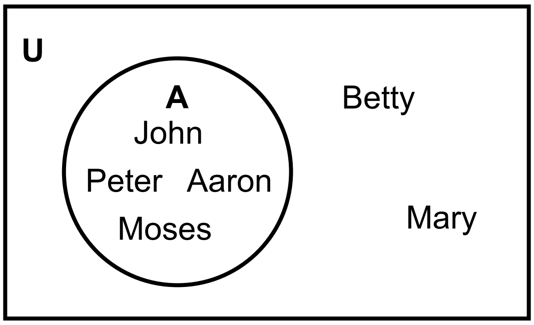
What is a Venn diagram?

Venn diagrams, sometimes called set diagrams are diagrams that show in practice possible logical relations between collections of sets. Venn diagrams were first used around 1880 by John Venn, an Englishman.

Today Venn diagrams are still used. The diagrams make difficult problems easy to understand.

A Venn diagram shows relationship between sets, by representing them as regions enclosed by simple closed curves.

Below is a Venn diagram showing a Universal set that is the rectangle denoted by **U**. Inside the universal set is a subset of the universal set. The subset denoted by **A**.



The Venn diagram shows Mr. Wai‟s children. He has 6 children of which 4 are boys.

Example 2

In this Venn diagram the rectangle represents names starting with the letter P.

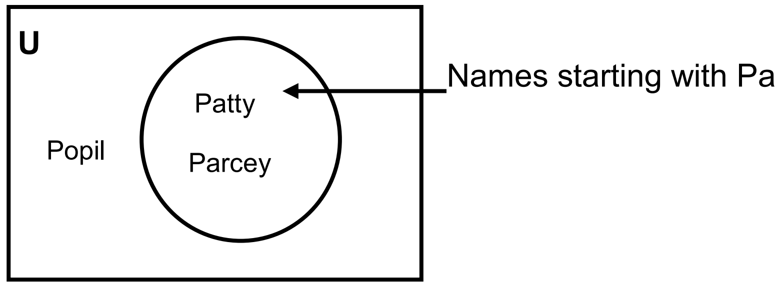
The names Popil, Patty and Parcy all start with the letter P, so they are in the rectangle.

However, Patty and Parcy start with Pa, so the two names are in the circle.

Popil doesn‟t start with Pa, it starts with Po and is not in the circle but are all in the rectangle (Names starting with P).

Turn to the next page and see how this information is shown on a Venn diagram.

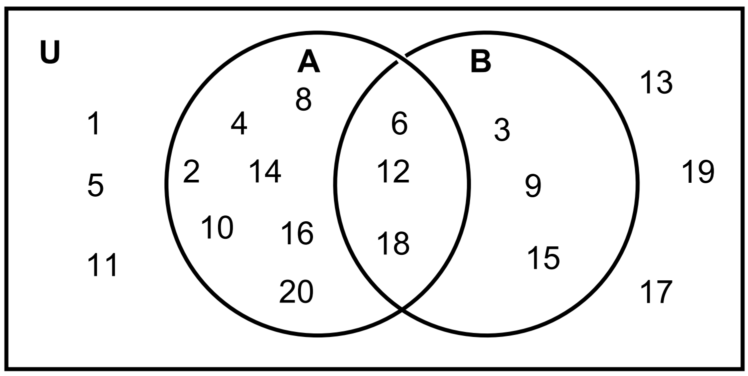
The Venn diagram shows students names starting with P



Popil, Patty and Parcy are all in a rectangle. They are in one set, they are names that start with P.

Example 3

In this Venn diagram the rectangle represents all the whole numbers from 1 to 20.



Each circle in the rectangle represents a result.

The result in circle A is all the multiples of 2.
Circle B is all the multiples of 3.
The parts of the circle that overlap show the results that are common to both.
This part is called the **intersection** of A and B

Anything not belonging to either of the two but that is still part of the set are placed outside the circles but within the rectangle.

The rectangle is called the **universal** set. It contains all the members possible including those in Set A and B.

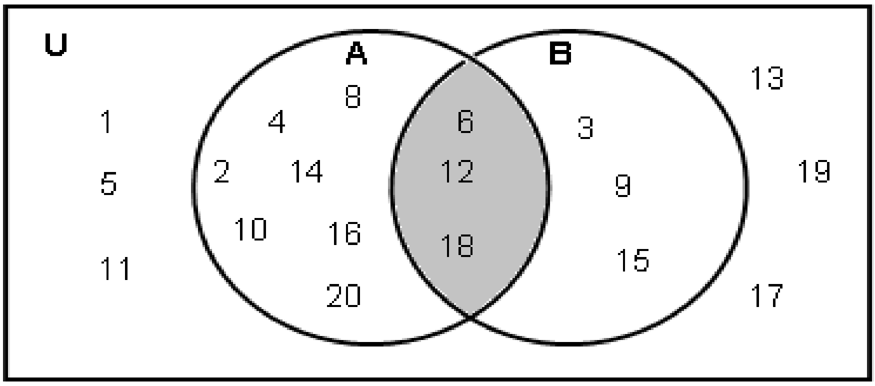
Represented in the Venn diagram are

U = The set of whole numbers 1 to 20

A = {multiples of 2} It is called a subset of U

B = {multiples of 3} it is also a subset of U

Multiples of 2 intersect with the multiples of 3 which is the set of multiples of 6. 6, 12 and 18 are the common multiples of 2 and 3.



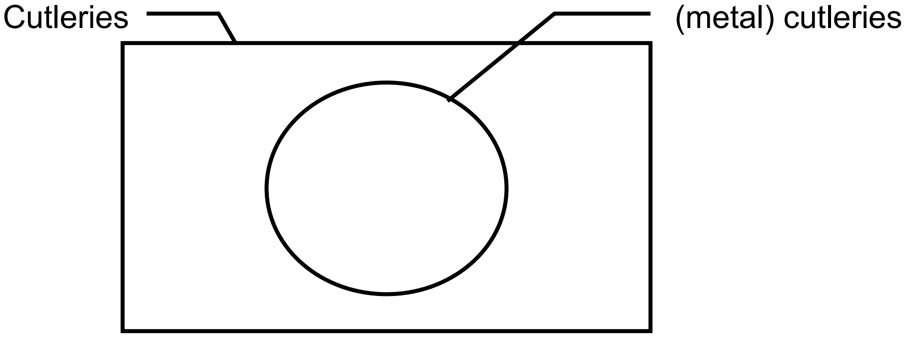
The shaded region on the Venn diagram shows the intersection of set A and B. It contains members that are both in set A and B. The set can also be called an **intersection** of set of A and B.

NOW DO PRACTICE EXERCISE 14

Practice Exercise 14

1. Place the following set of examples in the correct part of the diagram.

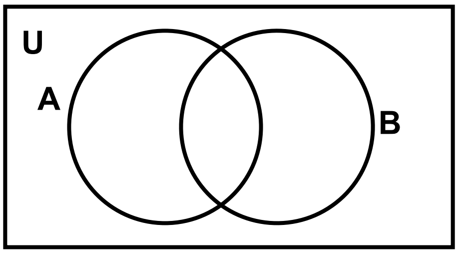
   (wooden spoon, butter knife, fork, bread knife)



2. Represent the set of whole numbers 1 – 20 in the correct part of the diagram.

   A: numbers greater than 8

   B: numbers that are multiples of 3

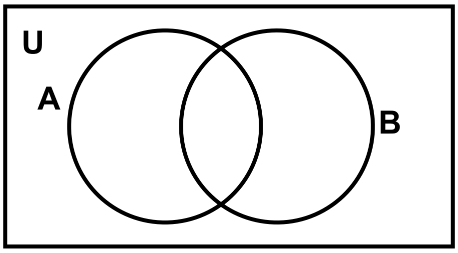


3. Represent the following sets on a Venn diagram

   U = {All counting numbers 1 to 30}

   A = {multiple of 2}

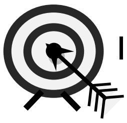
   B = {multiple of 5)



CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3

Lesson 15: Union and Intersections of Sets

In the previous lesson we learnt how to represent sets on a Venn diagram.



In this lesson, you will:

- describe the union and intersection of sets
- identify elements in a union and intersection of sets.

In this lesson, we are going to study Union and Intersection of sets.

Let us first study the meaning of the union of sets

A Union of two sets X and Y is the collection of all the members which are in Set X or Set Y without repetition. A union of X and Y is written as XUY. "U" is the symbol for union. It is read as X union B.

Example 1

If Set A = {1, 2, 3, 4} and Set B = {5, 6, 7, 8}

Then the union of set A and B is the members of Set A and Set B

It is written as AUB = {1, 2, 3, 4, 5, 6, 7, 8}

Example 2

If Set A = {pinkey, pongkey, donkey, monkey} and

Set B = {cow, pow, kow,low}

The union of set A and B is

AUB = {pinkey, pongkey, donkey, monkey, cow, pow, kow, low}

The set obtained by combining the sets A and B in this way is called the union of the sets **A** and **B**.

Sometimes members of a set are also members of another. How do we list a union of two sets? We obtain this set by first listing all the elements of A, and then the elements of B not in A.

Example 1

If A = (1, 2, 4, 6} B = {0, 11, 3, 4), what is the set A U B?

A U B = {1, 2, 4, 6, 0, 3}.

Notice we have combined the two sets A and B. Though 4 belong to both sets, we do not have to write it twice in the combined set.

Example 2

If Y = {2, 4, 6, 8, 10} and Z = {4, 8, 12, 16, 20}

Y U Z = {2, **4**, 6, **8**, 10, 12, 16, 20}

Notice that 4 and 8 belong to both sets. In a union of the two sets they do not have to be written twice.

Example 3

Set A = {Mary, Betty}

Set B = {Peter, Aaron, John, Moses, Mary, Betty}

AUB = {Mary, Betty, Peter, Aaron, John, Moses}

Mary and Betty were mentioned in both sets and were written once in the union set.

Example 4

Set A = {1, 2, 3} Set B = {3, 4, 5}

AUB = {1, 2, 3, 4, 5}

3 is a member of both sets and is written once in the union set.

INTERSECTION OF SETS

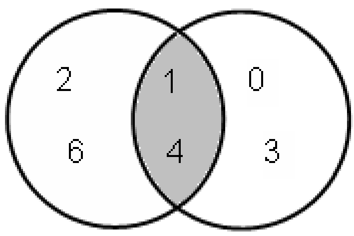
The intersection of two sets A and B, denoted by A∩B, is the set of all elements which belong to both A and B.

Example 1

If A = {1, 2, 4, 6} B = (0, 1, 3, 4), what is the set A ∩ B?

A∩B = {1, 4}

This can also be represented on a Venn diagram.



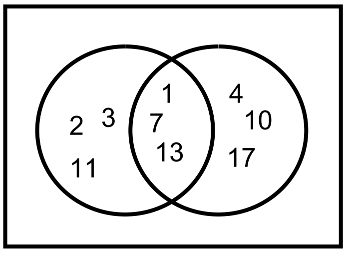
The shaded section of the Venn diagram represents the intersection of A and B.

Example 2

When Set A = {1, 2, 3, 7, 11, 13} and Set B = {1, 4, 7, 10, 13, 17},

then Set A∩B is all the common elements of Set A and B.

Therefore A∩B = {1, 7, 13}, this can be shown by using a Venn diagram as well



NOW DO PRACTICE EXERCISE 15

Practice Exercise 15

1. For each of the pairs of sets L and M below, list the elements of LUM.

(a)	L = {1, 3, 5}	M = {3, 5, 7}
(b)	L = {2, 4, 7}	M = {0, 2, 4}.
(c)	L = {2, 3, 4}	M = {2 ,4 8}.
(d)	L = {□, △}	M = {△, ○, ☆}.
(e)	L = {2, 3, 5}	M = {0, 1, 2}
(f)	L = {a, b, c}	M = {a, b, c, d}.
(g)	L = {1, 2, 3,…,100}	M = {0, 1, 2,…, 49}

2. Describe the union operation AUB in each of the following cases.

Example: A = {bus travellers}, B = {train travellers}.
AUB = {bus travellers, train travellers}.

(a) A = {Mathematics students}, B = {English Students}.

(b) A = {car drivers}, B = {truck drivers}.

(c) A = {men who jog}, B = {men who walk}

(d) A = {football players}, B = {cricket players}.

(e) A = {girls who like chocolates}, B = {girls who like ice-cream}.

3. For each of the pairs of sets L and M below, list the elements of L∩M.

(a) L = {1, 3, 5}, M = {3, 5, 7}.

(b) L = {2, 4, 7}, M = {0, 2, 4}.

(c) L = {2, 3, 4}, M = {2, 4 8}.

(d) L = {□, △}, M = {△, ○, ☆}.

(e) L = {2, 3, 5}, M = {0, 1, 2}

(f) L = {a, b, c}, M = {a, b, c, d}.

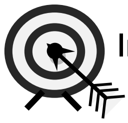
(g) L = {1, 2, 3, …, 100} M = {0, 1, 2,…, 49}

(h) L = {2, 4, 6, …}, M = {1, 3, 5,…}.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 3.

Lesson 16: Solving Problems with Venn Diagrams

In the last lesson, we learnt about intersection and union of sets.



In this lesson, we will:

- use Venn diagrams to illustrate and classify sets.
- solve problems from real life using Venn diagrams.

We learnt how to represent sets on Venn diagrams in lesson 14. In this lesson you are required to remember the concepts you learnt in the last lesson. It is therefore a good idea for you to revise the previous lesson notes.

Venn diagrams make is easy to solve problems. It will help you to clearly understand the problem.

Example 1

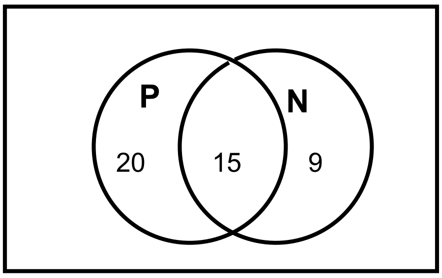
A paper boy delivers 35 copies of the Post Courier and 24 copies of the National in Taubi St. If he delivers both the Post Courier and National to 15 houses, to how many houses does he deliver?

a) The Post Courier and not the National?

b) The National and not the Post Courier?

c) Both papers?

To solve the problem using a Venn diagram, we must first draw a Venn diagram representing the problem, then we can answer the questions.

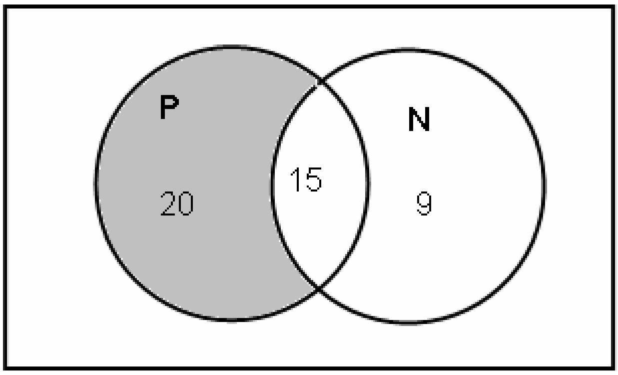


P is the number of houses that received the Post Courier and **N** is the number of houses that received the National Paper.

P∩**N** is the number of houses that received both the Post and national papers.

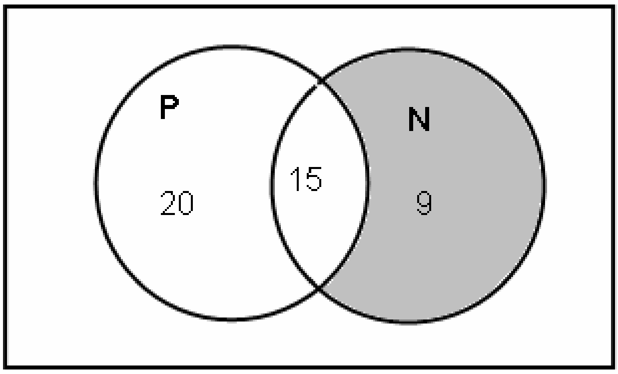
Turn to the next page to see how the question is answered.

a) The number of houses that received Post Courier and not the National.



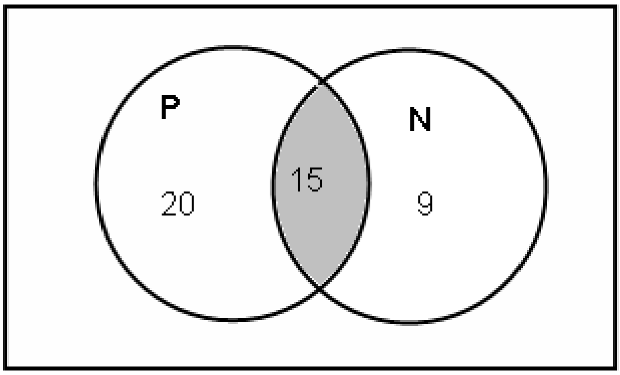
The shaded section of the Venn diagram shows the number of houses that received the Post Courier only. There are **20 houses** that got the Post courier only.

b) The number of houses that received the National and not the Post Courier



The shaded section shows the number of houses that received the National paper only. There are **9 houses** that received the national only.

c) The number of houses that received both the National and the Post Courier.



The section that is shaded shows the number of houses that received both the national and post.

We can see that **15 houses** received both the National and Post courier.

Example 2

Jinit sells 15 aigir plates and 14 mumu plates to her friends.If she sells both the aigir and mumu plates to 10 people then how many people will get the

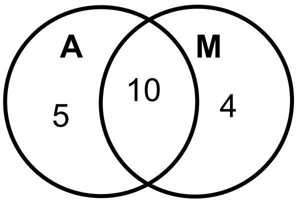
a) aigir and not the mumu?

b) mumu and not the aigir?

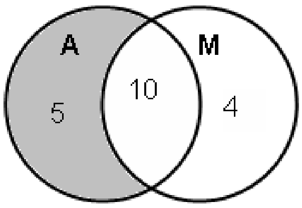
c) both mumu and aigir?

If we let **A** be the number of people who bought the aigir plates and
M be the number of people who bought the mumu plates.

Then the Venn diagram will look like this.

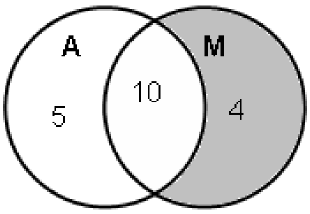


a) The shaded part shows the number of people who will get aigir and not mumu.



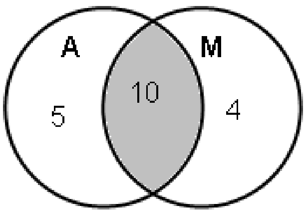
Five people will get aigir.

b) The shaded part shows the number of people who will get mumu and not aigir.



Four people will get mumu.

c) The shaded part of the Venn diagram shows the number of people who will get both aigir and mumu.



NOW DO PRACTICE EXERCISE 16

Practice Exercise 16

Use a Venn Diagram to solve the problem.

1. Students in a class play rugby or soccer or both. 15 play rugby, 10 play soccer and 5 play rugby and soccer. How many students

   a) play rugby but do not play soccer ______________________________

   b) play soccer but do not play rugby ______________________________

   c) play rugby or soccer but not both ______________________________

   d) are in the class ______________________________

2. Of a group of people at the market, 48 carry umbrellas and 52 wear raincoats, while 40 carry an umbrella and wear a raincoat. How many

   a) carry an umbrella or wear a raincoat ________________________

   b) carry an umbrella but do not wear a raincoat __________________

   c) wear a raincoat but do not carry an umbrella __________________

3. Of 65 table tennis players, 8 can play with the left hand but 3 of them can play with both the left and the right hand.

   a) How many can only play left-handed? ________________________

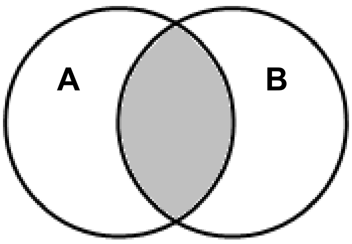
   b) How many can play with the right hand? ________________________

   c) How many can play right-handed? ________________________

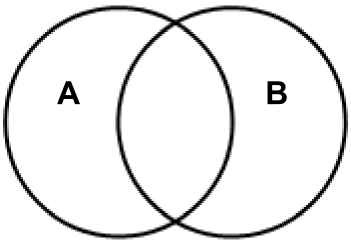
CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB STRAND 3

SUB-STRAND 3: SUMMARY

- A set is a collection of things or objects. Sets can consist of elements of various nature – people, physical objects, numbers, signs and others.
- The objects which belong to the set are called the elements or members of the set.
- The elements or members of a set can be listed in braces { }
- Sets may be defined by listing or describing the elements, and are often denoted by a capital letter.
- The empty set has no elements. This can be represented as { }. An empty set is also called a null set, it is denoted by Ø.
- The union of two sets A and B, denoted by $A \cup B$, is the set of all elements which belong to either A or B or both without repetition.
- The intersection of two sets A and B, denoted by $A \cap B$, is the set of all elements which belong to both A and B.
- A Venn diagram is a diagram showing the relationships between sets, by representing them as regions enclosed by simple closed curves.
- An intersection of set A and B represented by a Venn diagram can be shown as



- A union of set A and B can be represented by a Venn Diagram as



- Venn diagrams help us to visualise and solve problems.

REVISE LESSONS 13 – 16. THEN DO SUB-STRAND TEST 3 IN ASSIGNMENT 5

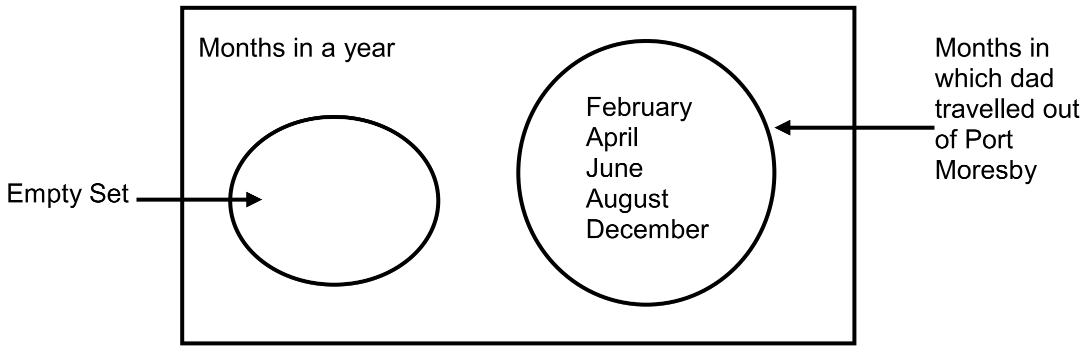
ANSWERS TO PRACTICE EXERCISE 13 – 16

Practical Exercise 13

1. (a) own investigation. See your teacher
   (b) own investigation. See your teacher
   (c) Central, Oro, Milne Bay, Gulf, Western, ESP, WSP, Madang, Morobe, New Ireland, Manus, WNB, ENBP, EHP, Simbu, WHP, Enga, SHP, NCD
   (d) Jan, Feb, Mar, Apr, May, Jun, Jul, Aug, Sep, Oct, Nov, Dec
   (e) Mathematics, science, language, Social Science, Making A Living, Personal Development.

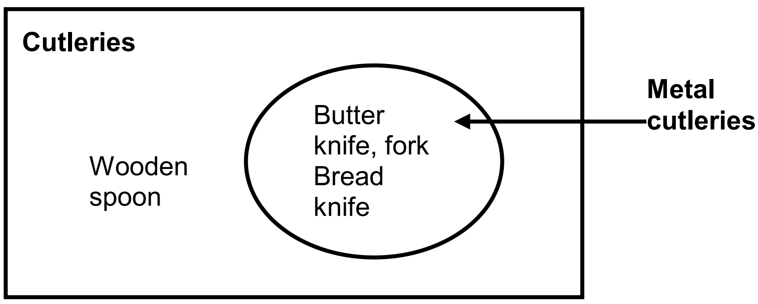
2. (a) they come under the set of types of fish
   (b) come under the set of energy food
   (c) come under the set of fruits
   (d) volcanoes in PNG
   (e) vowels in the alphabet

3. This is one answer to question 3. You can come up with your own.

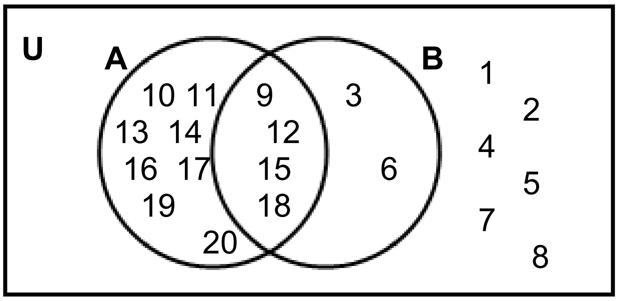


Practice Exercise 14

1.

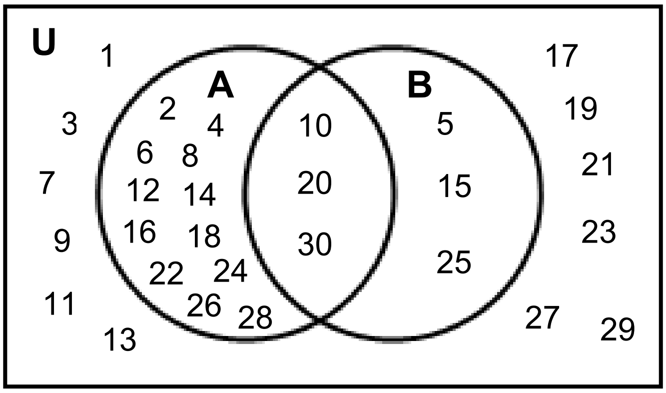


2.



U = {the set of whole numbers 1 - 20}
A = {numbers greater than 8}
B = {multiples of 3}

3



U = {the set of whole numbers 1 to 30}
A = {multiples of 2}
B = {multiples of 5}

Practice Exercise 15

1. (a) {1, 3, 5, 7}
   (b) {2, 4, 7, 0}
   (c) {2, 3, 4, 8}
   (e) {2, 3, 5, 0, 1}
   (f) {a, b, c, d}
   (g) {0, 1, 2, 100}

2. (a) A ∩ B = {students of Maths or students of English or both}
   (b) A ∩ B = {drivers of cars or trucks or both}
   (c) A ∩ B = {men who jog or walk or both}
   (d) A ∩ B = {players of football or cricket or both}
   (e) A ∩ B = {girls who like chocolates or ice cream or both}

3 a) {3, 5}
b) {2, 4}
c) {2, 4}
d) {△}
e) {2}
f) {a, b, c}
g) {1, 2,…., 49}

Practice Exercise 16

1. a) 10
b) 5
c) 15
d) 20

2. a) 60
b) 8
c) 12

3. (a) 5
(b) 57
(c) 60

END OF SUB-STRAND 3

SUB – STRAND 4

PROBABILITY

SUB-STRAND 4: PROBABILITY

Introduction

Dear student,

Welcome to sub-strand 4 of your Grade 8 Mathematics Strand 5. In this Sub-strand, you will extend your knowledge on probability and learn helpful and interesting facts about it.

Probability is a way of expressing knowledge or belief that an event will occur or has occurred.

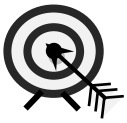
We use probability in many of the decisions we make in our daily lives. It is an interesting area in mathematics. Probability is used in many fields today such as in engineering, statistics, gambling and so forth.

In this sub strand, you will:

- re-visit some terms you studied in Grade 7 and learn some new terms.
- calculate probabilities of different problems in a real life situation
- explore the results of independent events
- calculate probabilities from individual events
- explore the chances of winning and losing games
- explore the social implications of probability.

Lesson 17: What are Chances?

You learnt to identify terms used to describe chance in your Grade 7 Mathematics Strand 5 Sub-strand 3.



In this lesson, you will:

- define the following terms: probability, outcomes, chance, events, sample space, likely event, unlikely event, simple or individual events and impossible events
- use suitable words to describe the chance of an event occurring in everyday life situations

First we will need to revise some of the old terms and study new terms in probability.

- **Chance** is the likelihood of something happening.
- **Probability** is defined as the measure of the likelihood of an event or outcome.
- **Outcome** is another word for result**.**
- **Event** is a particular outcome as a result of an experiment**.**
- **Sample space** is a method of listing possible outcomes of an event.
- **Simple** or **individual events** are events which consist of a single outcome.

The chance of something happening is always discussed in everyday conversation.

Examples

a) What is the chance of arriving at school on time on a PMV?

b) What is the chance of wearing my sports uniform on Thursday?

c) What is the chance of winning in a card game this afternoon?

There are words that describe chances.

An event that will not occur is called an impossible event. Impossible events have a probability of 0.

Examples of impossible events are

a) Wearing a red pair of socks when you have only white pairs of socks.

b) Wining a raffle ticket when you have no tickets.

c) Eating rice when you have only banana in your plate.

Sure and possible describe the certainty of something happening. Events that are certain to occur have a probability of 1.

Examples

a) Picking a red marble from a bag of red marbles.
b) Eating rice when you have rice in your plate
c) Using an umbrella during a rainy day.

- **Likely events have a high chance of occurring.**
- **Unlikely events have a low chance of occurring.**
- **Even chance events have a 50-50 chance of happening.**
- **All such events have a probability between 0 and 1.**

Example 1 Likely events

a) Picking a red marble from a box of marbles.
b) Winning a race after a month"s training.
c) Scoring the highest score in a test after a week"s study.

Example 2 Unlikely events

a) Picking a red marble from a box containing black marbles and one red marble.
b) Winning a race after a day"s training.
c) Scoring the highest score in a test after an hour"s study.

Example 3 Even chance events

a) Getting a head from a toss of a coin is a 50-50 chance event. There are two sides to the coin you either get a head or a tail.
b) Rolling a one from a six -sided die.
c) Picking a red marble from a bag containing 10 blue and 10 red marbles.

Here are some more examples.

a) A pencil box contains 100 coloured pencils. 80 are black, 15 are yellow, 4 are red and 1 is blue.

If you are blinded folded and are asked to choose from the box.
Black will have a high chance of being picked, because there are more black pencils than the other two colours.

Yellow, red and blue will have a low chance of being picked. Comparing yellow, red and blue pencils, yellow will have more chance of being picked than the other two. It is because yellow is more in number than the other two colour pencils.

The blue pencil has a very low chance of being picked compared to yellow and red.

b) Six students enter a lucky draw at a coca cola promotion.

July Jack Mary Noreen Jepta Jacob

Their names are written on a blank dice and then the dice is rolled. The name that appears on the top once rolled, wins the draw.

Who is most likely to win – a girl or July?

July has 1 in 6 chances of winning the first prize. The probability of July winning the prize is $\frac{1}{6}$.

There are three girls so there are 3 in 6 chances of a girl winning the first prize. The probability of a girl wining a prize is $\frac{3}{6}$.

We can say that a girl is more likely to win a prize than July.

c) If the following 9 balls are placed in a sack. What is the chance that a soccer ball is picked?

The chance of a soccer ball being picked is $\frac{4}{9}$ since there are 4 soccer balls from 9 balls in total.

Based on example c) What is the chance of picking a pink ball?

Picking a pink ball is an impossible event since there are no pink balls in the bag. Therefore, the probability of choosing a pink ball is 0.

Sometimes the probability of event can change over time.

Example

It is certain to be Friday tomorrow if today is Thursday.

NOW DO PRACTICE EXERCISE 17

Practice Exercise 17

1. A box contains 10 red marbles and 4 blue marbles. You are blind folded and are asked to pick a marble from the box.

   a) What colour of marble will you most likely choose?

   ____

   b) Which colour of marble is less likely to be picked?

   ____

2. Number cards are placed in a jar. The jar has cards numbered

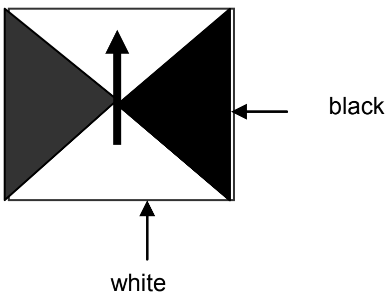
   1 1 3 2 5 6
   1 1 2 1 3 1 1

   a) Which number is likely to be picked?

   ____

   b) Which number has a low chance of being picked?

2. A rectangular spinner shown is spun.



   a) What is the chance of getting a black from a spin?

   ____

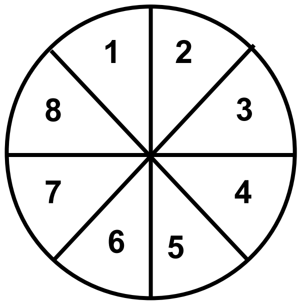
   b) What is the chance of not getting a black?

   ____

   c) What is the chance of getting white?

   ____

4. A number wheel is shown.



a) What is the chance of choosing a 10?
____

b) What is the chance of choosing a prime number?
____

5. 20 Blue, 10 pink and 10 green coloured tickets are placed in a box to be drawn.

a) What is the chance of picking a blue ticket?
____

b) What is the chance of picking a green or pink ticket?
____

c) What is the chance of not picking a blue ticket?
____

---

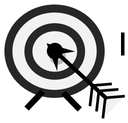
6. It is raining. What is the chance of wearing

a) sun glasses ______________________________
b) cold shirt ______________________________
c) track suits ______________________________
d) a gum boot ______________________________
e) rain coat ______________________________
f) sports wear ______________________________

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB -STRAND 4

Lesson 18: Probability of an Event

You have revised and defined the concept of chance in your Grade 7 work.



In this lesson, you will:

- describe the chance of an event using a suitable word like certain, impossible, unlikely, likely and 50-50 chance
- show the position of events on a number line according to the chance that it may occur.

In the last lesson we described chances using the words likely, unlikely, impossible and certain.

As a revision to the last lesson describe the following using impossible, likely, unlikely and certain.

1) Tomorrow will be Christmas.
2) I shall have kaukau to eat tonight.
3) It will be raining tomorrow.
4) I shall be in the United States of America tonight.
5) I shall be in bed by 10 pm tonight.
6) I will be at school at 10 am today.

Events that were sure to happen were described as **certain** to occur and such events have a probability of 1.

Example

Choosing a pair of black socks from a basket containing only black socks

Events that were never going to occur were described as **impossible** events and such events have a probability of 0.

Example

Choosing a pair of black socks from a basket containing only white socks

Events that have a 50-50 chance of occurring were described as having an **even chance** of happening.

Example

Getting a head in a toss of a coin

Events that are likely or unlikely to happen have a probability between 0 and 1.

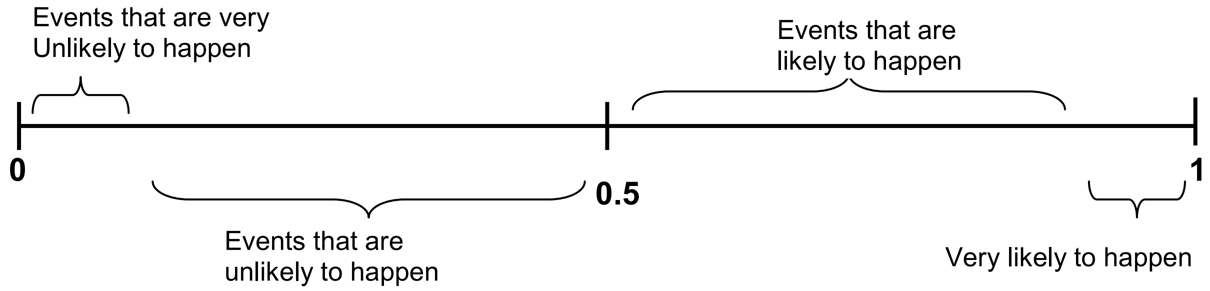
Events that are closer to 1 have a higher chance of happening.

There are events that can change over time.

Example

It is certain to be Wednesday tomorrow if today is Tuesday otherwise it is impossible.

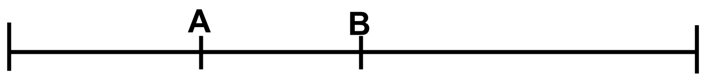
Events can be positioned on number lines according to its chance of occurring. Here is a number line showing how chances described as certain, likely, unlikely and impossible can be plotted.



We can summarise the number line above as:

- **A probability of 1 tells us that the event is sure to happen.**
- **A probability of 0 tells us that the event will NOT occur.**
- **All other probabilities between 0 and 1 may or may not occur.**
- **An event plotted to the right of another event on the number line has more chance of occurring.**
- **An event plotted to the left of another event on the number line has a less chance of occurring.**

Example 1

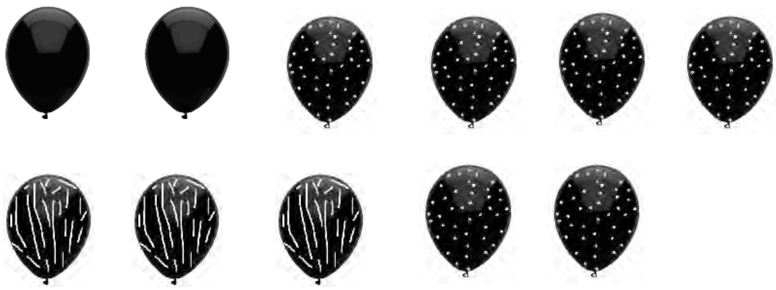


Event B is to the right of event A. Therefore event B has more chance of occurring than event A.

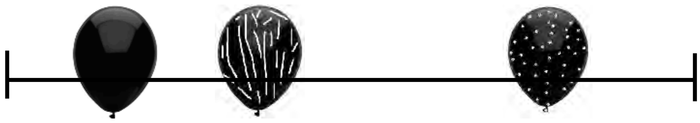
Turn to the next page and see more examples.

Example 2

You are blind folded and are asked to pick a balloon from the following.



The number line below shows the chance of each of the balloons shown to be picked.



The number line shows that a white spotted balloon is most likely to be picked while the white striped and black balloons are less likely to be picked.

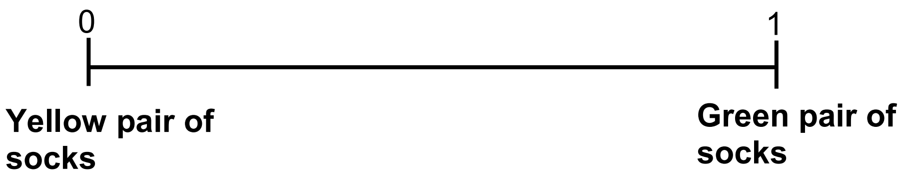
Comparing the black and white striped balloon, the white striped balloon has more chance of being picked than the black.

Example 3

A bag contains green pairs of socks.

a) What is the chance that a pair of yellow socks is picked?
b) What is the chance that a pair of green socks is picked?

The number line answers the two questions.



Picking a pair of yellow socks is an impossible event, because the bag contains only green socks, therefore picking a green pair of socks is an event that is sure to happen.

Sometimes an event is given a measure to describe its chances of not occurring.

Example 4

Given: Event A has a probability of $\frac{3}{4}$.

Event B has the probability of $\frac{1}{4}$.

Comparing the two events, event A has more chances of happening than event B.

We can use Decimal fractions to measure probability in the same way.

Example 5

Given: Probability of event A is 0.8

Probability of event B is 0.2

Event A has more chances of happening than event B because its probability is closer to 1.

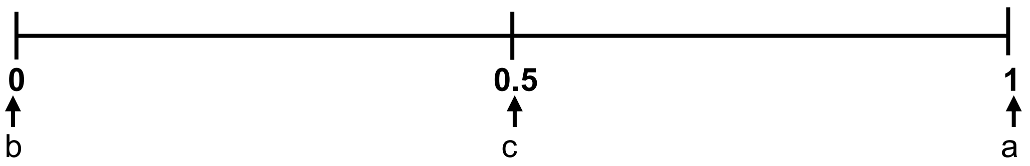
Example 6

Describe the chance of the events below occurring using a suitable word and then mark their positions on the number line.

(a) Choosing a red card from 10 red cards.
(b) Winning a raffle if you have no ticket.
(c) A new baby is a girl.

Answer: (a) Choosing a red card from 10 red cards is an event that is sure to occur.
(b) Winning a raffle if you have no ticket is impossible.
(c) A new baby is a girl is a 50-50 chance.

Here is a number line showing the descriptions.



In summary

- When you look at the value of a probability, you can predict its occurrence.
- The position of an event on the number line can help you predict its occurrence.

NOW DO PRACTICE EXERCISE 18

Practice Exercise 18

1. Compare the following three events and answer the questions that follow.

   Event A has probability of 0.2
   Event B has probability of 0.3
   Event C has probability of 0.5

   (a) Which event has an even chance of occurring?

   (b) Which event has less chance of occurring?

   (c) Which event has a low chance?

---

2. Describe the events below using a suitable word.

   (a) A rainy day will be cold.

   (b) A newborn baby will be a boy.

   (c) Choosing a white marble from a bag containing red and black marbles.

   (d) Throwing a tail when a coin is tossed.

   (e) Choosing a white sock from a drawer with 20 white socks.

   (f) Throwing a 5 when a die is tossed.

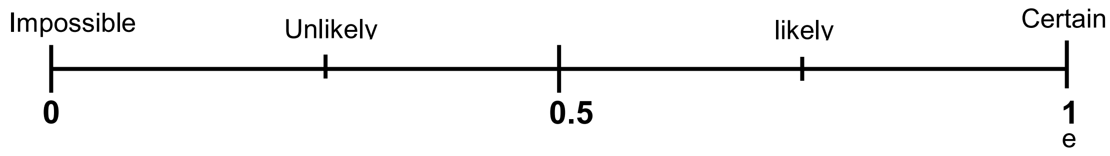
   (g) Choosing a black marble from a bag containing 5 black marbles and a red marble.

   (h) Throwing an even number when a die is tossed.

   (i) Throwing a head or a tail when a coin is tossed.

   (j) The sun will shine in Hagen on a December day.

3. Place the events in question 2 (a) to (j) on the number line between 0 and 1. e has been done for you



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4. For which of the following events would you give a probability of „O‟?

a) A person swimming faster than a shark.

b) Drawing a green card from a normal pack of cards.

c) Obtaining a „6‟ from one roll of a die.

d) The day immediately following Wednesday being Tuesday.

e) There will never be 29 days in a month.

f) You will grow shorter in the course of the coming year.

---

5. Describe the chance of each of the following events using likely, unlikely, impossible and certain.

a) A „head‟ from one toss of a coin.

b) A „tail‟ from one toss of a coin.

c) A birthday occurring on a Saturday.

d) The last digit of a telephone number being 7.

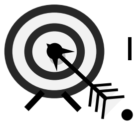
---

6. What does a probability of 0 mean?

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4.

Lesson 19: Simple and Individual Events

In the last lesson you described the chance of various events occurring and plotted the events on a number line according to their chance of occurring.



In this lesson, you will:

- write all possible outcomes in an individual event or throwing a die/dice.
- work out problems on simple or individual events.

What are simple or individual events?

Simple or individual events consist of a single outcome.

Examples of simple events are:

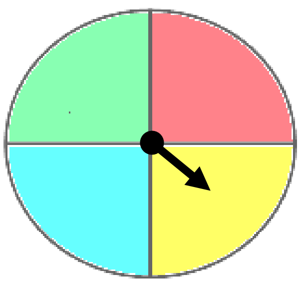
1. Tossing a twenty toea coin
2. Spinning a wheel
3. Rolling a die

In Grade 7, we learnt that the outcomes of events were the result of experiments and that the outcome of simple events can be listed.

Here are some examples of the listing of outcomes of simple events.

1. A bag contains a red, green, black and yellow marble. If a marble is picked the possible outcomes can be red, green, black or yellow.

2. Rolling a spinner once



The spinner has 4 colours red, green, blue and yellow, therefore outcomes are Green, Red, Blue or Yellow.

3. When tossing a coin, the outcomes are head or tail.

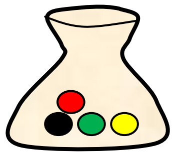
4. When rolling a die, the results can be a 1, 2, 3, 4 ,5 or 6.

In Grade 7, we learnt how to list all possible outcomes in a sample space. A sample space consists of open and closed brackets { } with all the possible outcomes listed inside. A sample space is denoted by the capital letter **S**.

The number of elements is written as n(s) where n stands for the number of elements in (**s**) the sample space.

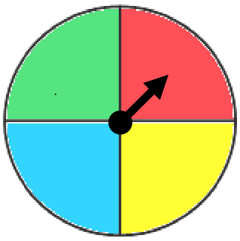
Examples

1. A jar contains a red, green, black and yellow marble.



S = {red, green, black, yellow} n(s) = 4

2. Roll the spinner once



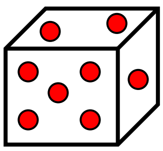
S = {green, red, blue, yellow} n(s) = 4

3. When tossing a coin, the outcomes are head or tail.

C = {head, tail} n(s) = 2

4. When rolling a six sided die, its possible outcomes can be listed in a sample space as this,

D = {1, 2, 3, 4, 5, 6} n(s) = 6



Here is a trial question for you to answer.

A bag contains a pair of red, blue, green, orange and white socks. You are blind folded and are asked to pick a pair of socks.

(a) List the sample space for the above event.

(b) How many elements are there in this sample space?

Here are the answers to the trial questions. Correct your work.

(a) S = {red, blue, green, orange, white}

(b) n(s) = 5

Calculating Probability

We have learnt that probability is the chance or likelihood that something will happen. It is the ratio of the number of ways an event can occur to the number of possible outcomes.

We will use the following model to help us calculate the probability of simple events.

$$\textbf{Probability} = \frac{\textbf{number of favourable outcome}}{\textbf{total possible outcomes}}$$

As you can see, with this formula, we will write the probability of an event as a fraction. The numerator is the number of chances and the denominator is the set of all possible outcomes.

Example 1

A coin is tossed. What is the probability that head turns up?

Before we start the solution take note that P(h) means the probability that head turns up. When you see P( ) this means to find the probability of whatever is indicated inside of the parenthesis.

Let us first identify the sample space

S = {head, tail} and n(s) = 2

n(s) = 2 becomes the denominator of the probability and there is one head in a coin and this is the favourable outcome.

Therefore, $\text{Probability} = \dfrac{\text{number of favourable outcome}}{\text{total possible outcomes}}$

$$P(h) = \frac{1}{2}$$

Example 2

What is the probability of scoring a 5 in a single roll of die?

There is only one 5 in a die. There is 1 chance or 1 favourable outcome. This becomes the numerator while there are 6 possible outcomes that is 1, 2, 3, 4, 5 or 6. or n(s) = 6 becomes the denominator.

Therefore $P(5) = \dfrac{1}{6}$

Example 3

What is the probability of drawing an ace from a deck of 52 playing cards, when the Deck of cards is cut once.

There are 4 aces in a pack of card so there are 4 chances, this goes to the numerator, the total of 52 cards are the total possible outcomes in other words n(s) = 52 goes to the denominator.

$$P(\text{ace}) = \frac{4}{52}$$

Cards are used in many games and we need to know what is found in a pack. A pack has 4 suits. These are

♠ Spades ♥ Hearts ♦ Diamonds ♣ Clubs

Each suit contains 13 cards. These are:

Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen and King giving a total of 52 cards

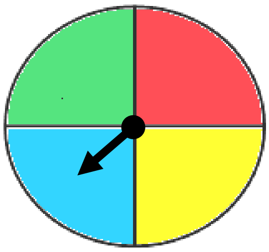
A pack of cards also contains a Joker but this is not in the count of 52.

NOW DO PRACTICE EXERCISE 19

Practice Exercise 19

List the outcomes of Questions 1 to 7.

1. Spinning one colour from a spinner.



2. Picking a certain coin from a purse that contains one each of 5, 10, 20, 50 toea coins.

3. The colour of the lights when you have to stop at a traffic light

4. 2 coins are tossed

5. A coin is tossed

6. A die is rolled

7. A coin is tossed and the arrow on the spinner in (1) is spun

8. Find the probability of the following events happening

   a) a coin landing on its head when it is thrown ________________

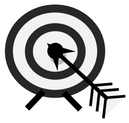
   b) a die landing on 6 when it is rolled __________________

   c) a green marble being picked from the jar containing one red, one green one yellow, one black, and one blue marble:

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4

Lesson 20: Percentage in Probability

In the last lesson you calculated probabilities of events.



In this lesson, you will:

- write probabilities as percentages, fractions and decimals.
- solve probability problems and express answers in percentage.

Probabilities can be written as fractions, decimals or percentages just like any other ordinary numbers. You have learnt how to change a quantity or a number from decimal to fractions to percentages or vice versa. The rule used in changing a number or quantity from a decimal to a fraction then to a percentage or vice versa is the same for probabilities.

Let us revise the rules.

- **To change a fraction to a decimal, divide the numerator by the denominator or by using equivalent fractions which have a denominator of a power of 10.**
- **To change a decimal to a fraction, divide the digits of the decimal number by a power of 10.**
- **To change a fraction or decimal to a percentage, multiply by 100.**

Examples

Decimal	Fraction	Percentage
0.8	$0.8 = \frac{8}{10} = \frac{4}{5}$	$\frac{4}{5} \times 100 = 80\%$
0.08	$0.08 = \frac{8}{100} = \frac{2}{25}$	$0.08 \times 100 = 8\%$
0.008	$0.008 = \frac{8}{1000}$ $= \frac{1}{125}$	$\frac{1}{125} \times 100 = \frac{4}{5}\ \%$
1.25	$1.25 = \frac{125}{100}$ $= 1\frac{1}{4}$	$1.25 \times 100 = 125\%$

You have learnt that probabilities are measured between 0 and 1. A probability of 0 is a measure of an impossible event and has a 0% chance of occurring, while a probability of 1 has a 100% chance of occurring. As we have learnt on page 137 a measure of probability can be written as a fraction, a decimal or a percentage.

Examples

1. Write the probability of $\frac{1}{2}$ as a decimal.

   Solution: $\frac{1}{2} = 1 \div 2 = 0.5$

2. Write the probability of 0.5 as a fraction.

   Solution: $0.5 = \frac{5}{10} = \frac{1}{2}$

3. Write the probability of 0.5 as a percentage.

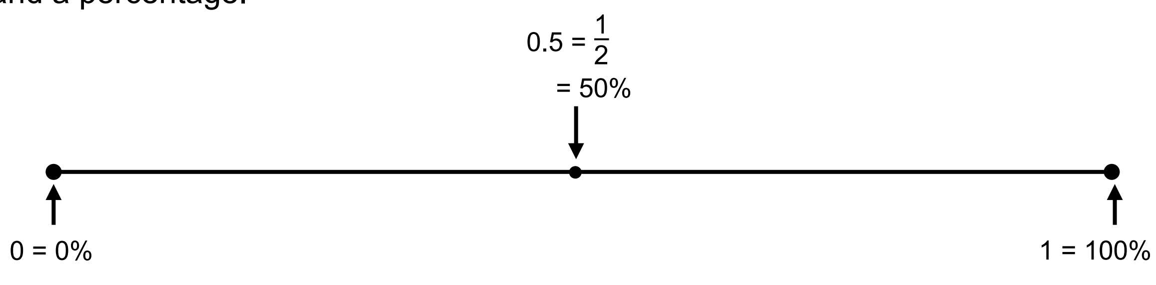
   Solution: $0.5 \times 100 = 50\%$

4. Write the probability of $\frac{1}{2}$ as a percentage.

   Solution: $\frac{1}{2} \times 100 = 50\%$

The four examples show how probabilities can be written as a fraction, decimal or a percentage. The examples also show a measure of even - chance events, such events have 50% chance of occurring which is the same as 0.5 or $\frac{1}{2}$.

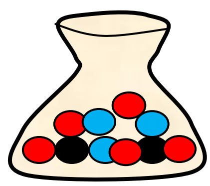
Here is a number line showing equivalent probabilities written as a fraction, a decimal and a percentage.



Let us now study some examples of solving problems involving probability.

Example 1

A bag consists of 10 marbles. 5 are red, 3 are blue and 2 are black.



a) What percentage of the marble is red?

   Solution: Five out of 10 marbles are red, therefore $\frac{5}{10} = \frac{1}{2} = 50\%$

b) What is the chance that a red marble is picked?

Solution: The bag consists of 10 marbles. The 10 marbles are the 10 possible outcomes.

$$P(\text{red}) = \frac{\text{favourable outcome}}{\text{total possible outcomes}}$$

$$= \frac{5}{10}$$

$$= \frac{1}{2} = 50\%$$

There is a 50% chance of picking a red marble.

c) What is the chance of choosing a blue marble?

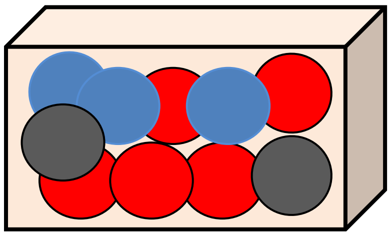
$$P(\text{blue}) = \frac{\text{favourable outcome}}{\text{total possible outcomes}}$$

$$= \frac{3}{10} = 0.3 = 30\%$$

There is a 30% chance of choosing a blue marble.

Example 2

A box contains 10 balls. 2 black, 3 blue and 5 red.



The probability of choosing a black ball is 0.2.

Solution: $P\text{ (Black)} = \frac{2}{10}$ $2 \div 10 = 0.2$

$= 0.2$

The probability of choosing a blue ball is 0.3.

Solution: $P(\text{blue}) = \frac{3}{10} = 0.3$

Example 3

A family consists of 3 females and 2 males. What is the chance of selecting a female from the group?

Solution: P(female) $= \dfrac{\text{Number of females in the family}}{\text{Total number of family members}}$

$= \dfrac{3}{5}$

$= 0.6$

$= 0.6 \times 100 = 60\%$

This means that 60% of family member are females.

Example 4

a) A lolly bag contains 3 red, 2 yellow and 1 orange lolly. If 1 lolly is selected find the chance that it is red.

P (red) $= \dfrac{3}{6}$

$= \dfrac{1}{2} = 0.5$

0.5 can be changed into a percentage and that is $0.5 \times 100 = 50\%$

This means that 50% of the lollies are red.

b) Find the probability that the lolly selected is not a yellow lolly.

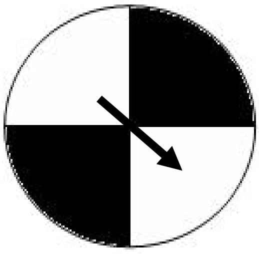
P (not yellow) $= \dfrac{4}{6}$

$= \dfrac{2}{3}$

NOW DO PRACTICE EXERCISE 20

Practice Exercise 20

1. What is the probability, as a fraction, that the spinner will stop on black?



2. Write the probability in Question 1 as a percentage.

3. Use the diagram of the spinner in Question 1 to give the probability, as a fraction and as a percentage, that the spinner

   a) will stop on

      (i) white (ii) black

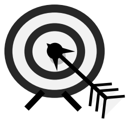
   b) what answer do you get when you add all the two probabilities.

4. What is the probability that the spinner will stop on red?

CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUBS-TRAND 4

Lesson 21: Making a Systematic List

In the last lesson, you learnt how to calculate the probability of an event.



In this lesson, you will:

- use the method of systematic listing in getting the outcomes of an event.

What is a systematic List?

A systematic list is a list of outcomes of experiments or surveys arranged in a manner that is easy to read.

Outcomes of experiments can be arranged systematically using a

a) frequency table
b) grid
c) Tree Diagram

We will list outcomes using a frequency table and a grid reference in this lesson. The tree diagram will be studied in the next lesson.

How can we use a frequency table as a method of systematic listing?

We learnt how to use a frequency table in sub-strand 2; the skill learnt will be extended to listing outcomes of experiments.

Example The following are outcomes of tossing a die 20 times.

1	1	2	3	4	4	3	2	5	6
4	2	2	6	5	6	6	6	5	4

The results are shown in the table below using the frequency distribution table method of sub-strand 2.

Outcome	Tally	Frequency	Probability
1	II	2	$\frac{2}{20} = \frac{1}{10}$
2	IIII	4	$\frac{4}{20} = \frac{1}{5}$
3	II	2	$\frac{2}{20} = \frac{1}{10}$
4	IIII	4	$\frac{4}{20} = \frac{1}{5}$
5	III	3	$\frac{3}{20}$
6	~~IIII~~	5	$\frac{5}{20} = \frac{1}{4}$
	Total tosses	20	1

The shaded row tells us that, in the twenty throws of the die 2 appeared 4 times. The probability is $\frac{4}{20}$.

From the table we can now read the probabilities of each outcome easily. We can use the rule below to find the probability to confirm our answer.

$$\textbf{Probability of 2} = \frac{\textbf{Frequency}}{\textbf{Total number of toss}} = \frac{\mathbf{4}}{\mathbf{20}}$$

Example 2

The following are results of tossing a coin 30 times.

H	T	T	H	H	H	H	T	H	H
T	T	T	H	T	T	H	T	H	H
H	T	T	T	H	H	T	T	T	H

The results are now placed in a Frequency table.

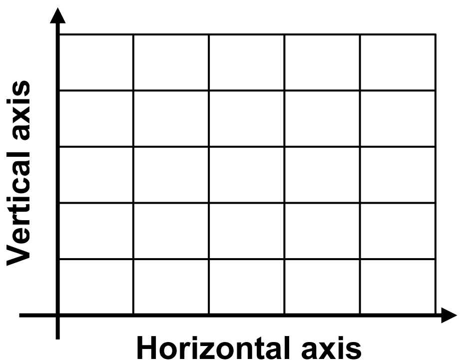
Outcome	Tally	Frequency	Probability
Head(H)	卌 卌 卌	15	$\frac{15}{30} = \frac{1}{2}$
Tail(T)	卌 卌 卌	15	$\frac{15}{30} = \frac{1}{2}$
	Total toss/Probability	30	$\frac{30}{30} = 1$

The shaded row shows that head appeared 15 times out of the 30 tosses and the probability is $\frac{1}{2}$.

Another way of listing outcomes is called the **Grid**. A grid is used when two events are carried out simultaneously.

The concept of using a grid is similar to plotting points on a map grid which you studied in Strand 4.

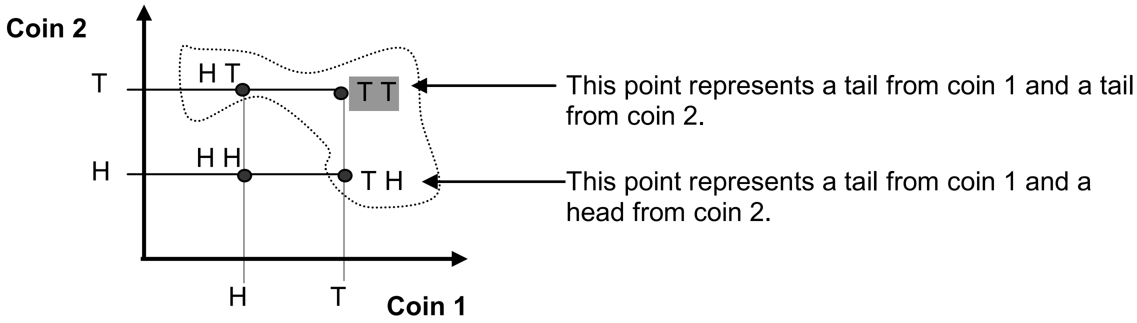
In a grid, there are two axes, the horizontal axis and the vertical axis.



To name the point: The value on the horizontal axis is named first followed by the value on the vertical axis.

Example 1

Two coins are tossed once.



The four points on the grid tells us that there are 4 possible outcomes.

By reading the grid we can answer the following questions.

1. How many points on the grid show a tail from both coins?

   Answer: One point that is the shaded point on the grid

2. What is the probability of getting both tails?

   Answer: $\text{Probability} = \dfrac{\text{Number of points}}{\text{Total Number of points on the grid}} = \dfrac{1}{4}$

3. How many times did tail appear?

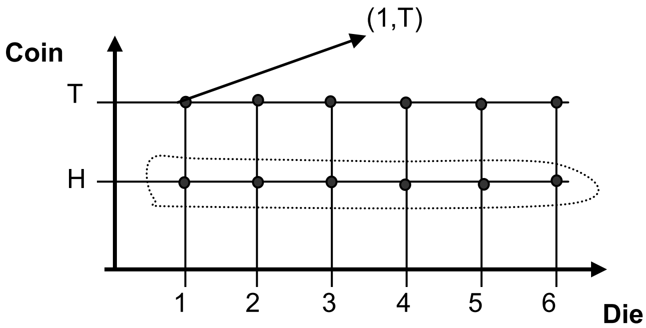
   Answer: The 3 points bounded by the curved line tells us the number of times tail appeared. Therefore our answer to this question is tail appeared 3 times.

4. What is the probability of getting heads?

   Answer: There are 3 points representing heads therefore head appeared 3 times.

Example 2

The outcomes of a coin and a die tossed simultaneously.



The points on the grid tell us that there are 12 possible outcomes.

By reading the grid, these questions can be answered.

1. What is the probability of tossing a head?

   Answer: the bounded 6 points out of 12 points represent heads therefore the probability is $\frac{6}{12} = \frac{1}{2}$

2. What is the probability of getting a tail?

   Answer: there are 6 points out of 12 points on the grid that represent tail therefore the probability is $\frac{6}{12} = \frac{1}{2}$

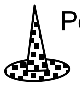
3. What is the probability of getting a tail and 1?

   Answer: There is only one point shown by an arrow on the grid that represents the point. Therefore the probability is $\frac{1}{12}$

NOW DO PRACTICE EXERCISE 21

Practice Exercise 21

1. When dropping a pin, it either lands with the point up (u) or the point down (d)


Point up (u)


Point down (d)

The following are the results of the event.

d u d d d d u d u d u d d u d

Present the results on a frequency table and use the table to answer questions that follow.

Outcomes	Frequency	Probability
u		
d		
Total		

a) What is the probability of the pin landing with the pointer up?

b) What is the probability of the pin landing with the pointer down?

---

2. When a plastic cup is dropped it either lands with the open end up (o) or the bottom end up (b).

The frequency Table shows the results of the experiment. Study the table to answer the questions that follow.

Outcomes	Frequency	Probability
o	15	
b	20	
Total	35	

a) Complete the probability column

b) Work out the probability of the cup landing with the open end up

c) Work out the probability of the cup landing with the bottom end up

d) Calculate the probability of the cup landing with the bottom end down

e) What is the probability of the cup landing with the open end down?

f) What is the total probability?

3. Draw a grid showing outcomes of two dice thrown simultaneously and list the outcomes in a sample space

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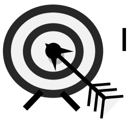
CHECK YOUR WORK. ANSWERS ARE AT THE END OF SUB – STRAND 4

Lesson 22: Tree Diagrams

In the last lesson, you learnt how to get the outcomes of an event, using systematic listing.



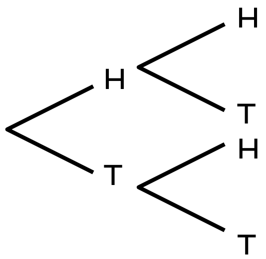
In this lesson, you will;

- define tree diagrams
- use the tree diagram method to get the outcomes of an event.

First, let us define tree diagrams.

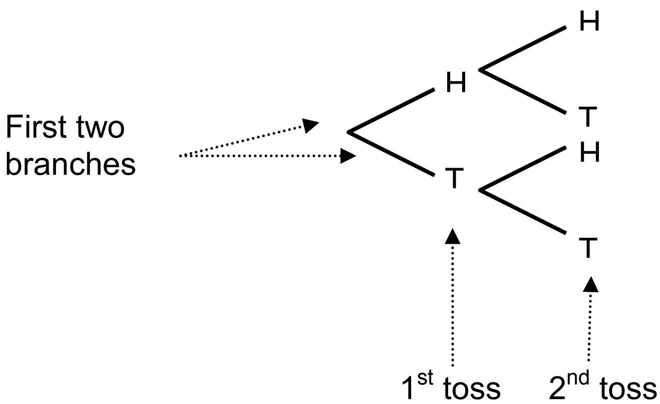
A tree diagram is a systematic way of listing all possible outcomes in an experiment and is also used to calculate probabilities.

This is how a tree diagram looks like. The diagram has branches showing outcomes .



Each branch represents one possible outcome. Tree diagrams are usually used to find the probabilities of 2 or more events happening.

The tree diagram below is showing outcomes of tossing a coin twice. Let us study the Tree diagram.



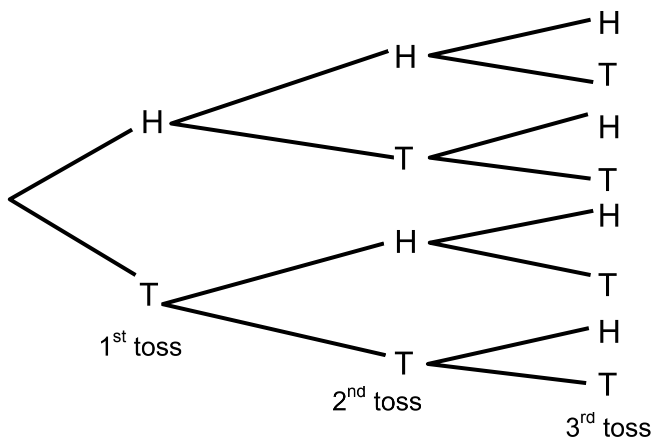
The first two branches represent the possible outcome of the first toss.

After the first toss we can then get a head or tail on the second toss so we draw two branches after each of the outcomes from the first toss.

The list of all possible outcomes is obtained by following every possible path on the tree branches.

The outcomes are {HH, HT, TH, TT}. There are 4 possible outcome, the branches representing the last toss indicate the number of outcome.

Example 2 A coin is tossed 3 times



The first two branches represent the possible outcomes of the first toss, head or tail. After the first toss we can then get a head or tail on the second toss so we draw two branches after each of the outcomes from the first toss. We repeat this for the third toss. The list of all possible outcomes is obtained by following every possible path through the tree.

The possible outcomes are

{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

Here are some questions that can be answered by studying the tree diagram.

a) How many possible outcomes are there?

{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

There are 8 possible outcomes. The branches representing the 3rd toss indicate the total number of outcomes.

b) What is the probability of getting all heads in the three tosses?

{<u>HHH</u>, HHT, HTH, HTT, THH, THT, TTH, TTT}

There is only one chance out of 8 possible outcomes (underlined)
Therefore the probability is $\frac{1}{8}$. We can also use the rule we learnt in the last Lesson.

$$P(HHH) = \frac{\text{favourable outcomes}}{\text{Total possible outcomes}} = \frac{1}{8}$$

c) What is the probability of getting two tails?

{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

The chances of getting two tails is 3 out of 8 possible outcomes, therefore the probability of getting two tails is $\frac{3}{8}$. We can use the rule to work out the probability.

$$P(2T) = \frac{\text{favourable outcomes}}{\text{Total possible outcomes}} = \frac{3}{8}$$

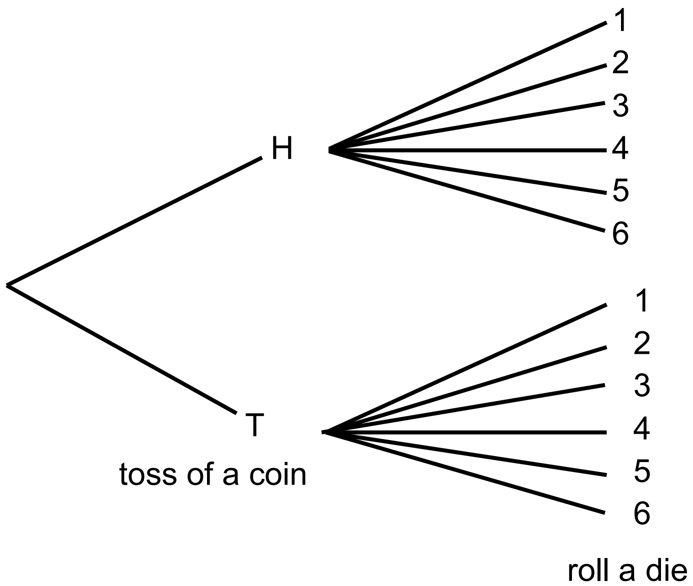
d) What is the probability of getting two heads?

{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

There are 3 favourable outcomes out of 8. The probability is $\frac{3}{8}$.

$$P(2H) = \frac{\text{favourable outcomes}}{\text{Total possible outcomes}} = \frac{3}{8}$$

We will now work out the total number of outcomes of tossing a coin and rolling a die.



There are 12 possible outcomes. They are:

H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6

The following are some questions that can be answered from the tree diagram.

a) What is the probability of getting a 1?

H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6.

The probability is $\frac{2}{12} = \frac{1}{6}$

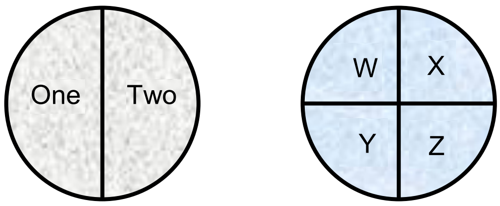
b) What is the probability of getting a tail?

From the outcomes: H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6. There are 6 possible chances of getting a tail, therefore the probability is $\frac{6}{12} = \frac{1}{2}$.

We can also use the rule to work out the probability

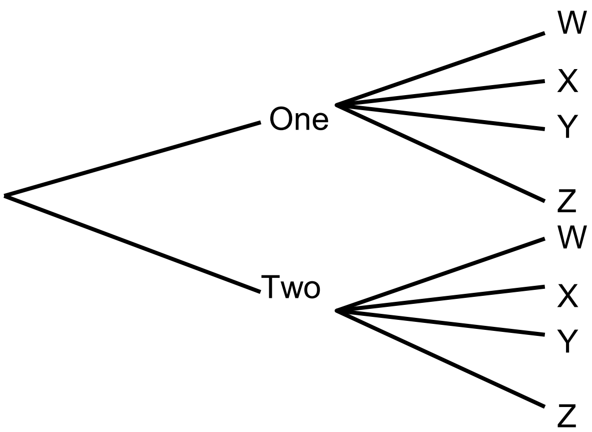
$$P(\text{tail}) = \frac{\text{favourable outcomes}}{\text{Total possible outcomes}} = \frac{6}{12} = \frac{1}{2}$$

Here is another example on spinning two wheels



1^{st} **wheel** 2^{nd} **wheel**

Here are the outcomes on a tree diagram



From the tree diagram of the two spinners there are 8 possible outcomes.

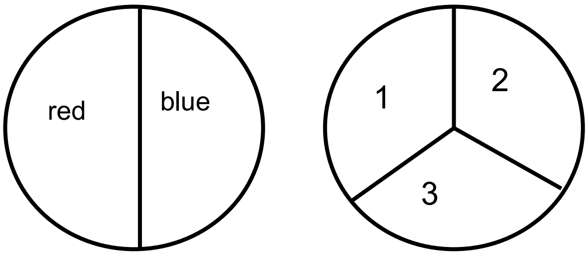
They are {One W, One X, One Y, One Z, Two W, Two X, Two Y, Two Z}

The probability of getting one is $= \frac{\text{favourable outcomes}}{\text{Total possible outcomes}} = \frac{4}{8} = \frac{1}{2}$

NOW DO PRACTICE EXERCISE 22

Practice Exercise 22

1. For the two spinners below draw a tree diagram showing all the possible outcomes.



Use your tree diagram to find the probability of getting,

(a) 1 and a red.

(b) 2 and a blue.

(c) an odd number and blue.

(d) a number less than 2 and red.

---

2. Three coins are tossed.

(a) Draw a tree diagram to illustrate this and list all the possible outcomes.

(b) Use the tree diagram to work out the probability of getting,

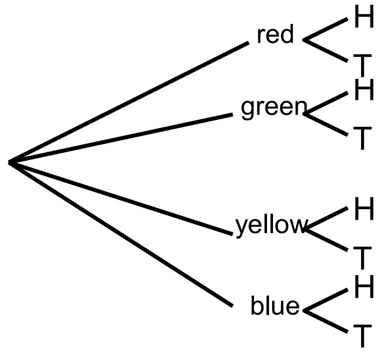
(i) 3 tails.

(ii) 1 tail and 2 heads.

(iii) 2 tails and 1 head.

(iv) no tails.

3. Here is a tree diagram showing the outcomes of a spinner with four sections coloured red, green, yellow, blue and a coin being tossed.



Use the tree diagram to work out the probability of:

(a) red on the spinner and a head

(b) yellow on the spinner and a tail

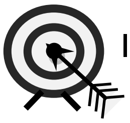
(c) green on the spinner and a head

(d) red on the spinner and tail.

CHECK YOU WORK. ANSWERS ARE AT THE END OF THE SUB–STRAND 4

Lesson 23: Social Implications of Probability

In the last lesson, you learnt about systematic methods of getting the outcomes of an event.



In this lesson, you will

- identify the social implications of probability.

First let us discuss the importance of chance in various life situations.

Many things are a mixture of something you can predict and something that depends on chance.

Here are some examples.

1. You can predict that the rain will not fall for weeks due to the dry season but by chance, it may.

2. You can predict the yam harvest to be plentiful next year as a result of the increased number of gardens but some things may occur by chance such as a prolonged dry season.

Think about mixing some things you can predict and something that depends on chance.

Write your thoughts here.

In the same way chance also applies to gambling. Using probability, you can understand the reality of the chances you have in gambling and making wise decisions.

Example

1. In a game of dart at the market, the prize of hitting the bull‟s eye is 1 litre of Coke. This is how the game is played.

   On a dart board, there are 22 numbers. The numbers range from 1 to 20, 25 and the bulls eye which is valued at 50. When you pay a fee of 50t, the operator gives you three darts to attempt hitting the bull to win the prize.

There are 22 possible outcomes of throwing a dart. The probability of hitting a bullseye on the first attempt is $\frac{1}{22}$ that is there is only one bullseye on a dart board therefore there is 1 out of 22 chances so the probability is $\frac{1}{22}$.

Three attempts at hitting the bullseye is a probability of $\frac{3}{22}$. In this game the probability of winning is unlikely, however people like to win so they keep trying.

The reality of gambling is simple. The more you spend, thinking that you are increasing your chances of winning, the more money you lose.

At the end the operator of the game benefits and the gambler loses.

You should now understand the fact that in gambling, whether it is a card game, rolling dice, lottery tickets, poker machines, horse racing, the chances of winning is very low while the risk taken is very high.

Look at the following social consequences of gambling.

1. You can become addicted to it and as a result

    - you will have less or no money to save
    - it will affect your budget prioritizing, whether to spend money on needs or to gamble.
    - It will lead to borrowing money and leave you in debt.

    These are some consequences of gambling in the family.

2. The quality of living standards today depends on how much money we have. When gambling affects the income, family needs are not met. As a result:

    - there is domestic violence
    - children go hungry
    - basic needs are not met e.g. clothes, lunch, bus fares.

NOW DO PRACTICE EXERCISE 23

Practice Exercise 23

Read about the game on bingo below and answer the questions that follow.

About 30 or 40 people play the game of Bingo. Each one gets a card with about 20 different numbers written on it. Each person has a different selection of numbers. The operator of the game pulls out numbers that are randomly chosen. As he calls out the numbers, people mark the numbers on their card if it is called out. The first person who has all his or her numbers called out cries “Bingo!” and that person wins the game.

Suppose there are 40 people playing the game, the chance of winning is 1 in 40. The operator gains whatever the difference is between the cost of the prize and the takings of the players. If the prize is a chicken, and the players pay 50 toea each (40 x 50t = K20), and a chicken costs 15 Kina, then the operator gains 5 kina, the winner gains K14.50 and the losers lose 50 toea each.

Use the example above to answer the questions according to each given situation?

1. There are 40 people playing. If the prize is a 5kg bag of rice that costs K25 and the players pay K1 each;

   (a) What are the chances of winning?

   (b) How much does the operator gain?

   (c) How much does the winner gain in value?

2. There are 100 people playing. Each player pays K1.50. The prize is a jersey that costs K40.00

   (a) What are the chances of winning?

   (b) How much does the operator gain?

   (c) How much does the winner gain in value?

   (d) Is it worth the risk when you know that your chances are $\frac{1}{100}$ that is most unlikely?

3. A raffle for a mobile phone valued at K49 was drawn after all 50 tickets were sold out for K3 each.

   (a) What is the probability of winning?

   ____

   (b) Who gains the most?

   ____

   (c) How much does he gain?

   ____

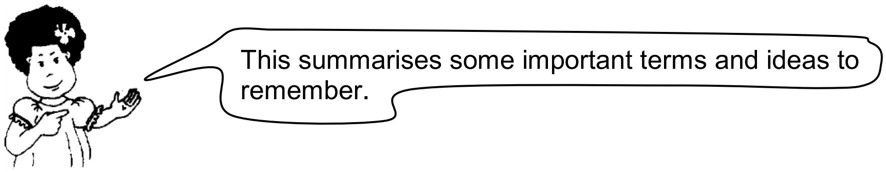
   (d) Think about this.

   Why gamble when we know the risks we are taking.

NOTE! Think twice before gambling.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF SUB-STRAND 4

SUB-STRAND 4: SUMMARY



- The chances of an event occurring is always discussed in everyday situations.
- There are many words used to describe the chance of something happening. Some words used in this Strand are like, likely, 50 – 50 chance, possible, impossible, certain.
- Events that have a 100% chance of occurring are certain to occur and have a probability of 1.
- Events that do not have a chance to occur are impossible events and have a probability of zero.
- Events that have a 50 – 50 chance of happening are even chance and have a probability of 0.5.
- Probabilities of other events fall between zero and 1.
- The Probability of any simple event is given by the formula $\dfrac{\textbf{favourable outcome}}{\textbf{total possible outcomes}}$.
- An Equally likely event is an event which has an equal chance of occurring.
- Probabilities can be expressed as a percentage, fraction or a decimal.
- A systematic list is a list of outcomes of experiments arranged in a manner that is easy to read.
- The types of systematic list used are the Frequency Table and Grid.
- The table of Frequency is a table similar to the table of Frequency used in Statistics. It can have 4 columns one for the outcome, tally, Frequency and Probability.
- A grid is used when two events are performed simultaneously. A grid consists of horizontal and vertical lines. Points on the grid represent the outcome of events.
- Probabilities of events can be read from a frequency table or the grid.
- A Tree Diagram is also a systematic way of presenting outcomes. It can also be used to work out probabilities.
- Gambling is a game of chance.
- Consequences of gambling include spending all your money, domestic violence, no lunch money or bus fare for children if the father is addicted to gambling and there are many more.

REVISE LESSONS 17-23. THEN DO SUB-STRAND TEST IN ASSIGNMENT 5

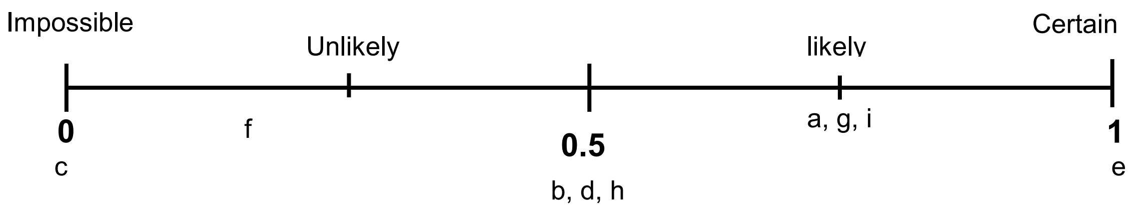
ANSWERS TO PRACTICE EXERCISES 18 - 23

Practice Exercise 17

1. (a) red (b) blue
2. (a) one (b) five and six have a low chance.
3. (a) even chance. (b) even chance. (c) even chance.
4. (a) no chance. (b) even chance.
5. (a) even chance. (b) likely. (c) unlikely.
6. (a) impossible (b) likely. (c) likely.
   (d) likely. (e) likely. (f) unlikely.

Practice Exercise 18

1. (a) Event C (b) Event A (c) Event B
2. (a) likely (b) 50 -50 (c) impossible
   (d) 50 -50 (e) certain (f) unlikely
   (g) likely (h) 50 -50 (i) certain
   (j) likely
3.



4. b, d and f.
5. (a) likely (b) 50 – 50 (c) likely
   (d) likely
6. An impossible event.

Practice Exercise 19

1. The outcomes are Green, Red, Blue or yellow.
2. 5, 10, 20 or 50 toea coins
3. red, orange or green
4. HH, TT, TH or HT
5. H, T
6. 1, 2, 3, 4, 5, 6
7. H Green H red, H blue, H yellow, T Green,T red,T blue or T yellow
8. (a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{1}{5}$

Practice Exercise 20

1. It is an even chance. P(black) = $\frac{1}{2}$

2. $\frac{1}{2} \times 100\% = 50\%$

3. (a) i) $\frac{1}{2}$ ii) $\frac{1}{2}$

   (b) the result is 1.

4. It is an impossible event, because the spinner has two colours black and white and therefore the event has a probability of 0.

Practice Exercise 21

1.

Outcomes	Frequency	Probability
u	**5**	$\frac{5}{15}$
d	**10**	$\frac{10}{15}$
Total	**15**	**1**

(a) $\frac{5}{15} = \frac{1}{3}$ (b) $\frac{10}{15} = \frac{2}{3}$

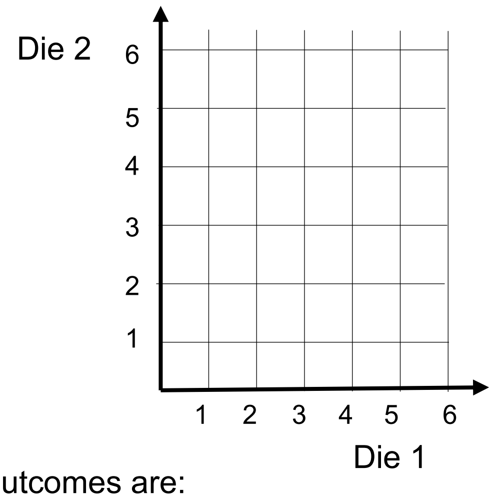
2.

Outcomes	Frequency	Probability
o	15	$\frac{3}{7}$
b	20	$\frac{4}{7}$
Total	35	**1**

(b) P(o) = $\frac{3}{7}$ (c) P(b) =$\frac{4}{7}$ (d) P(0) =$\frac{3}{7}$

(e) P(b)= $\frac{4}{7}$ (f) P(o) + P(b) = $\frac{3}{7} + \frac{4}{7} = 1$

3.

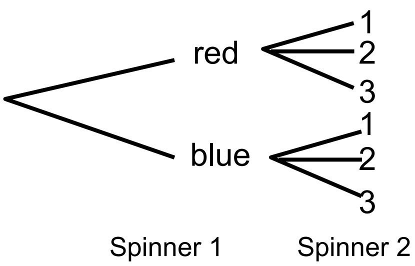


List of outcomes are:

{1 1, 1 2, 1 3, 1 4, 1 5, 1 6, 2 1, 2 2, 2 3, 2 4, 2 5, 2 6, 3 1, 3 2, 3 3, 3 4, 3 5, 3 6, 4 1, 4 2, 4 3, 4 4, 4 5, 4 6, 5 1, 5 2, 5 3, 5 4, 5 5, 5 6, 6 1, 6 2, 6 3, 6 4, 6 5, 6 6}

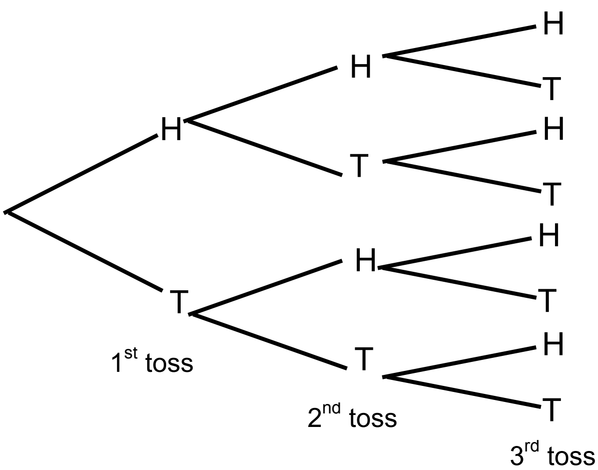
Practice Exercise 22

1.



(a) $\frac{1}{6}$ (b) $\frac{1}{6}$ (c) $\frac{1}{6}$ (d) $\frac{1}{6}$

2. (a)



(b) (i) $\frac{1}{8}$ (ii) $\frac{1}{4}$ (iii) $\frac{1}{4}$ (iv) $\frac{1}{8}$

3. (a) $\frac{1}{8}$ (b) $\frac{1}{8}$ (c) $\frac{1}{8}$ (d) $\frac{1}{8}$

Practice Exercise 23

1.	(a)	$\frac{1}{40}$	(b)	K15.	(c)	K24.		
2.	(a)	$\frac{1}{100}$	(b)	K110.	(c)	K38.50	(d)	No.
3.	(a)	$\frac{1}{50}$	(b)	operator	(c)	K101.		

END OF SUB STRAND 4

NOW YOU MUST COMPLETE ASSIGNMENT 5.
RETURN IT TO THE PROVINCIAL COORDINATOR.

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